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Final Report On

The Launching and Landing of Carrier Aircraft

Contract ONR 583 (01)
December 1952

Part II
Of Four Parts

By

A University of Kansas Research Group
University of Kansas
Lawrence, Kansas

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[redacted]

LIMITATIONS OF CABLE-DRIVE CATAPULTS

Part II of the Final Report on

THE LAUNCHING AND LANDING OF CARRIER AIRCRAFT

Contract ONR 583 (01)

December 1952

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By

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Lawrence, Kansas

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PREFACE

In August 1951 a University of Kansas Research group was assigned to study the general problem of the launching and landing of carrier aircraft. The work was done under contract ONR 583 (01). The purpose of the study was to obtain from a well-trained diversified group not too imbued with past and present Navy thinking and procedure, an independent evaluation of the problem and possible methods of solution, emphasis being placed upon development to meet future needs rather than just to solve immediate problems.

It was left to the group to choose those aspects of the problem on which to concentrate. As a result, certain aspects of the problem have been studied intensively while others have been considered only superficially. In analyzing the problem and dividing it into its several aspects, the group asked two questions:- (1) Is this aspect of the problem of decided importance? (2) Can the group make a worthwhile contribution by studying intensively this aspect of the problem? Emphasis was placed upon those aspects for which the answer to each question was affirmative.

The group submits its final report in four parts. The title and general content of each part is as follows:

Part I. General Report.

This section presents in a comprehensive yet understandable manner the problem as the group sees it, and makes clear what the group believes can and/or should be done. This section is relatively free of details but comprehensive as regards general conclusions.

Part II. Limitations of Cable-Drive Catapults.

This section presents a detailed study of the limitations of cable-drive catapults and the relative effects of different modifications of cable drives. It is rather analytical.

Part III. A Multi-Jet Driven Catapult (Hydrapult).

This section presents the results of a study of a multi-jet catapult which the group refers to as a "hydrapult." Although emphasis is placed upon the general features and operation of the proposed hydrapult, numerous details are included.

[redacted]

Part IV. Barricades.

This section presents the results of a model study of barricades. It contains many tabular data giving force distributions among the various elements of typical barricades. Numerous photographs are included.

The University of Kansas Research Group assigned to study this problem and submit this report was composed of the following staff members:

W. J. Argersinger, Jr., Ph.D., Associate Professor of Chemistry.

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W. M. Simpson,[1] Ph.D., Professor of Aeronautical Engineering and Chairman of department.

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LIMITATIONS OF CABLE-DRIVE CATAPULTS

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INTRODUCTION

The hydraulic drive (H-type) catapult with indirect connection to the aircraft by means of a cable has been used by the Navy for a number of years. Although this type of catapult has been quite successful, it seems to be the general opinion that it has been extended to a point near its ultimate practical capacity and that further expansion to higher capacities is unwise. NAMC Report No. M-4805, "Comparison of Hydraulic and C-Type Catapults," indicates that the weight of the H-type catapult increases very rapidly as the launching velocity approaches 275 to 300 knots. The study shows clearly that the limitations are due in large part to the presence of accelerated cable in the system, and to a smaller extent to the accelerated crosshead and sheaves.

Inasmuch as the indirect-drive cable catapult has advantages not offered by any direct-drive catapult yet proposed, it was deemed desirable by this group to investigate in some detail the origin and magnitude of the factors giving rise to these limitations, and to determine the approximate velocities beyond which cable-drive catapults are not feasible.

In the analyses of the following paragraphs, many simplifications have been made in order to bring out clearly the salient features of the indirect-drive system. In these analyses the effects of friction, plane thrust, shuttle weight, and cable vibration, as well as certain other less important factors, have all been neglected. While these factors may be vital for design purposes, they are not generally responsible for the inherent limitations of cable-drive systems.

In the following analyses, the type of engine has been unspecified. Whether the driving engine be hydraulic, slotted-tube or other type, is of no consequence except insofar as it alters the cable system. In all cases, however, it has been assumed that the moving engine parts are accelerated and decelerated by means other than cable-transmitted forces.

ELEMENTARY CONSIDERATIONS

Preparatory to a discussion of particular cable-drive catapults, it will be advisable to consider certain more general features of the problem. This section will therefore be devoted to a consideration of wire rope characteristics, a discussion of the acceleration of a straight cable, a treatment of the acceleration of a straight cable with attached masses, and a discussion of the effects of fixed and of moving sheaves.

Wire Rope Characteristics

The most important variable affecting the performance of indirect-drive catapults is the ratio of the safe working load of the cable to the cable linear weight. The weight of a given size of cable is readily found from tabulated data, but the safe working load must be obtained by dividing the breaking strength by an arbitrary factor of safety. The safety factor is dependent upon the type of service insofar as bending stress, abrasion, corrosion, heat, speed, and nature and degree of maintenance are concerned; it is also dependent upon the useful length of life desired. All of these factors which affect rope performance should be considered, but the most important is probably the bending stress that results when the rope is passed over a sheave or drum.

Although bending stress has been the subject of a great deal of investigation, there has not yet been found any satisfactory method of computing its magnitude. The 1947 edition of the Roebling Handbook* lists eight formulas for the computation of bending stresses, but the results computed from these equations vary greatly, the smallest being only 8.5 per cent of the largest. Obviously with this wide range of estimates available, it is impossible to compute rope bending stress with any degree of confidence.

It is recommended by wire rope manufactures that allowance for bending stress be absorbed directly into the safety factor. Since a stress caused by bending appears to have more effect on the rope life because of fatigue than it does on its useful static load, manufacturers state minimum sheave diameters that may be used for reasonable service life. The use of smaller sheaves than the recommended minimum will cause a greatly shortened life; larger sheaves will give a longer life. It should be noted that in the application to catapults, rope life is secondary in importance to high performance. Safety may be assured by a rigid and regular inspection procedure.

In the computations to follow, the ratio of working strength to linear weight has been based on the data shown in the following table. The table lists the breaking strength in pounds, the linear weight in pounds per foot, and the calculated breaking strength to linear weight ratio in feet for two types and for several sizes of wire rope. From this table it can be seen that all of the cables listed have a ratio of breaking strength to linear weight of about 55,000 ft. In later calculations, a factor of safety of approximately 5.5 has been assumed. Hence, the ratio of working strength to linear weight is 10,000 ft.; in other words, the weight of 10,000 ft. of rope hanging freely will cause the maximum safe working stress to occur in the upper

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* Published by John A. Roebling's Sons Company.

end. The use of a somewhat different factor of safety would alter the quantitative results of these sections but would not change the basic conclusions.

STRENGTH OF WIRE ROPE

Nominal Diameter, Inches	Approx. Linear Weight, w, lbs/ft.	Nominal Breaking Strength, B, lbs.	Calculated B/w, ft.
	6 x 19 with fiber core*		
1	1.60	95,000	59,400
1-1/4	2.50	146,000	58,400
1-1/2	3.60	208,000	57,800
1-3/4	4.90	280,000	57,100
2	6.40	350,000	54,700
	6 x 19 with independent wire rope core#		
1	1.76	102,100	58,000
1-1/4	2.75	157,000	57,100
1-1/2	3.96	223,600	56,500
1-3/4	5.39	301,000	55,800
2	7.04	376,300	53,500

* MIL-R-6015 Military Specification Rope; 6 by 19, Extra-High-Strength Wire (For Aircraft Launching and Arresting).

\# MIL-R-7871 Military Specification Rope; Extra-High-Strength Wire, 6 by 19 with 7 by 7 Independent Wire Rope Center (For Aircraft Launching and Arresting).

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Note that the maximum attainable velocity is independent of the acceleration so long as the maximum permissible load is applied continuously to the cable. If a constant acceleration of 4g is given to the cable, the distance required to attain a velocity of 802 ft/sec. is 2500 ft. This is also of course the maximum permissible length of cable.

On board ship, the accelerating distance is not the critical length, but rather the sum of the accelerating and decelerating distances is critical. Theoretically it is possible to stop the cable in practically zero distance without breakage by gripping the cable at every point on and in the cable. In practice, however, this situation is difficult to approach closely.

Case II:- Probably the simplest way to decelerate the cable is to replace the towing force by a braking force applied to the rear end of the cable. The braking force as well as the towing force must not exceed the safe working load of the cable. If a distance L is available to accelerate and then decelerate to zero velocity a cable of length L and linear weight w lbs/ft., it is obvious that the maximum velocity that can be attained without exceeding the safe working load of the cable will be that obtained with equal accelerating and decelerating distances. In other words, the safe working load is applied to the front end of the cable as a towing force for the first half of the distance L, and then the towing force is replaced by a braking force of equal magnitude for the last half of the distance L.

For such an arrangement, with $a = 10{,}000/L$, the maximum velocity attainable is

$$v = \sqrt{2\ a\ g\ (L/2)} = 567 \text{ ft/sec.} \qquad (3)$$

$$= 387 \text{ mph.}$$

$$= 336 \text{ knots.}$$

The same total shuttle run is required in Case II to reach a given terminal velocity as is required in Case I to reach a terminal velocity greater by a factor of $\sqrt{2}$.

If, in the future, materials with a higher strength to linear weight ratio are available, higher maximum velocities will be possible for any given system. An improvement by a factor of 4 in the strength to weight ratio would increase the maximum terminal velocity by a factor of 2.

<u>Acceleration of a Straight Cable with Attached Masses</u>

In any indirect-drive catapult, a great amount of mass in addition to that of the cable must be accelerated and perhaps decelerated by cable-transmitted forces. During the acceleration

run, the plane and shuttle as well as certain other components must be accelerated along with the cable; during the deceleration run, there may be a portion of the moving mechanism that must be decelerated by cable-transmitted forces.

Let it be assumed that a plane of weight W lbs. and an additional weight W_a lbs. are attached to the cable during acceleration, and that a weight W_d lbs. is attached to the cable during deceleration. Let the total length of run be L ft., and let a straight length of cable of length fL ft. be used to attach the towing cable to the towing engine. The system is shown in Figure 1. Two cases are considered:- Case I, in which the entire length L is used for acceleration, and Case II, in which the length L is divided into an accelerating portion A and a decelerating portion D. In Case II, the cable and attached mass W_d undergo a deceleration of dg ft/sec^2.

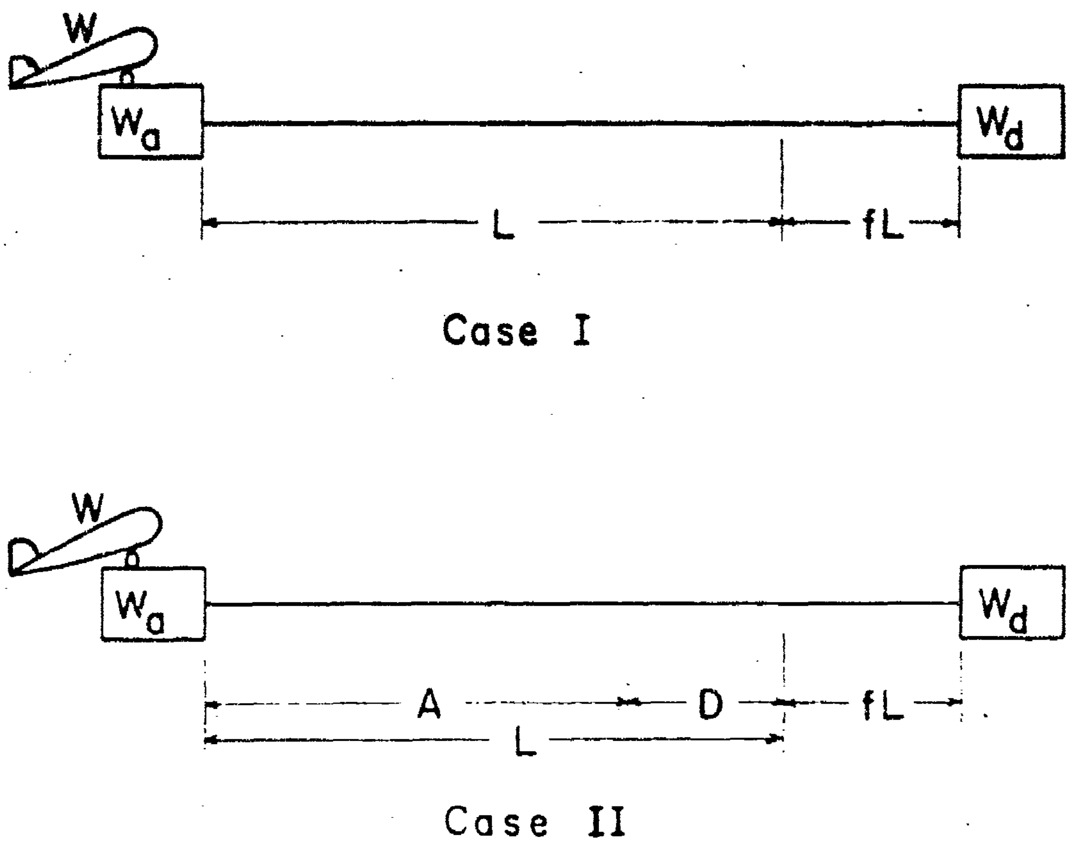


Figure 1. Straight cable with attached masses.

Case I:- With the notation defined in this and the preceding section,

$$P = \left[w\ (1 + f)\ L + W + W_a\right]\ a \quad . \tag{4}$$

At the end of the accelerated run of length L, the velocity v is given by

$$v = \sqrt{2\ a\ g\ L} = \sqrt{\frac{2\ (P/w)\ g\ L}{(1 + f)\ L + W/w + W_a/w}} \quad . \tag{5}$$

It is convenient at this point to introduce the concept of the equivalent length of these additional masses. Let the equivalent length of a given mass be defined as that length of cable which has the same weight as the mass being considered. Thus the total effective length L_e is given by

$$L_e = (1 + f)\ L + W_a/w + W/w \quad , \tag{6}$$

and the terminal velocity v by

$$v = \sqrt{gP/w}\ \ \sqrt{2\ L/L_e} = 567\ \sqrt{2\ L/L_e}\ \text{ft/sec.} \tag{7}$$

The terminal velocities which can be obtained with various values of the ratio $L_e/2L$ are shown in the table below; P/w is taken as 10,000 ft.

$\frac{L_e}{2L}$ or $\frac{L_a + L_d}{2L}$	v_{max}		W
1	567 ft/sec.	336 knots	
2	401	237	1,820 lbs.
5	254	150	12,600
10	179	106	30,600
20	127	75	66,600
50	80	47	175,000
100	57	34	355,000

Case II:- During the acceleration run

$$P = \left[w\ (1 + f)\ L + W_a + W\right]\ a \tag{8}$$

and at the end of the acceleration run, the terminal velocity v is given by

$$v = \sqrt{2\ a\ g\ A} = \sqrt{\frac{2\ (P/W)\ g\ A}{(1 + f)\ L + W_a/w + W/w}}\ . \tag{9}$$

During deceleration

$$P = \left[w\ (1 + f)\ L + W_d\right]\ d \tag{10}$$

and if the velocity is to be reduced to zero at the end of the decelerated run of length D = (L - A), then

$$v = \sqrt{2\ d\ g\ D} = \sqrt{\frac{2\ (P/w)\ g\ (L - A)}{(1 + f)\ L + W_d/w}}\ . \tag{11}$$

From equations (9) and (11) one finds

$$\frac{A}{L} = \frac{(1 + f)\ L + W_a/w + W/w}{2\ (1 + f)\ L + W_a/w + W/w + W_d/w} \tag{12}$$

which may be substituted in equation (9) to yield

$$v = \sqrt{\frac{P}{w}\ g}\ \sqrt{\frac{2\ L}{2\ (1 + f)\ L + W_a/w + W_d/w + W/w}}\ . \tag{13}$$

It is important to note that in this equation the weight of cable accelerated, w (1 + f) L, the weight of the cable decelerated, w (1 + f) L, the weight of the mass which must be accelerated, W_a, or decelerated, W_d, by cable-transmitted forces, and the weight of the plane, W, all enter in exactly the same manner. It is thus important that decelerated masses as well as accelerated masses be kept at a practical minimum.

If the effective lengths of cable during acceleration and deceleration are defined respectively as

$$L_a = (1 + f)\ L + W_a/w + W/w$$

and

$$L_d = (1 + f)\ L + W_d/w \quad , \tag{14}$$

then equation (13) becomes

$$v = \sqrt{\frac{P}{w}\, g}\ \sqrt{\frac{2\ L}{L_a + L_d}} = 567 \sqrt{\frac{2\ L}{L_a + L_d}}\ \text{ft/sec.} \tag{15}$$

The terminal velocities which can be obtained with various ratios of $(L_a + L_d)/2L$ are shown in the table on page 8; P/w is taken as 10,000 ft. This table contains calculated plane weights W for an illustrative example of Case II, in which L = 200 ft., f = 0.5 and $W_a = W_d = 0.05$ W. These are included merely to indicate the trend of plane weights with increasing values of the parameter $(L_a + L_d)/2L$. Note finally that for either Case I or Case II, the initial entry for $L_a/2L = 1$ corresponds to acceleration of the cable without plane or other attached mass.

Effects of Fixed and of Moving Sheaves

The effect of a sheave depends upon whether the sheave is fixed or moving, and upon whether it is a part of a reeved system. This section will be concerned with the effects of sheaves under various circumstances.

General Considerations

In the treatment of sheaves, the rotational and translational effects are considered independently of each other. In the subsequent analyses, the following notation will be used with reference to Figure 2 on the following page.

F_1 = Upper rope load (in lbs.)

F_2 = Lower rope load (in lbs.)

F_3 = Applied force (in lbs.)

v_1 = Velocity of the upper rope (in ft/sec.)

v_2 = Velocity of the lower rope (in ft/sec.)

v_3 = Translational velocity of the sheave (in ft/sec.)

a_1 = Acceleration of the upper rope (in g)

a_2 = Acceleration of the lower rope (in g)

a_3 = Translational acceleration of the sheave (in g)

w = Linear weight of the rope (in lbs/ft.)

W_s = Weight of the sheave (in lbs.)

W_y = Weight of the yoke (in lbs.)

r = Mean radius of curvature of the cable (in ft.)

k = Radius of gyration of sheave (in ft.)

ω = Angular velocity of sheave (in rad/sec.)

α = Angular acceleration of sheave (in rad/sec^2.)

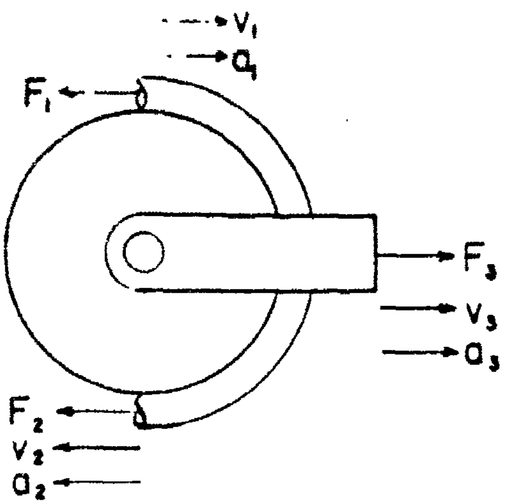


Figure 2. Forces, velocities and accelerations in a fixed or a moving sheave.

From the geometry, the following relations are apparent:

$$v_3 = \frac{v_1 - v_2}{2} \tag{16}$$

$$a_3 = \frac{a_1 - a_2}{2} \tag{17}$$

$$\omega = \frac{v_1 + v_2}{2r} \tag{18}$$

$$\alpha = \frac{a_1 + a_2}{2r} \tag{19}$$

Considerations of the rotational motion give:

$$F_2 - F_1 = \left[W_s \frac{k^2}{r^2} + w \pi r \right] \frac{a_1 + a_2}{2} \tag{20}$$

Considerations of the translational motion give:

$$F_3 = F_1 + F_2 + \left[W_s + W_y + w \pi r \right] \frac{a_1 - a_2}{2} \tag{21}$$

<u>Fixed Sheaves</u>

For a fixed sheave, $v_1 = v_2 = v$, $a_1 = a_2 = a$, $v_3 = 0$ and $a_3 = 0$. Then from equation (20)

$$\frac{F_2}{w} - \frac{F_1}{w} = \left[\frac{W_s}{w} \frac{k^2}{r^2} + \pi r \right] a \quad . \tag{22}$$

In this equation, the term in brackets on the right side corresponds to the sum of the actual length of cable in a 180° bend plus the equivalent length of the sheave itself. If the latter is designated by L_s, so that

$$L_s = \frac{W_s}{w} \frac{k^2}{r^2} \quad , \tag{23}$$

then equation (22) becomes

$$\frac{F_2}{w} - \frac{F_1}{w} = \left[L_s + \pi r\right] a \quad . \tag{24}$$

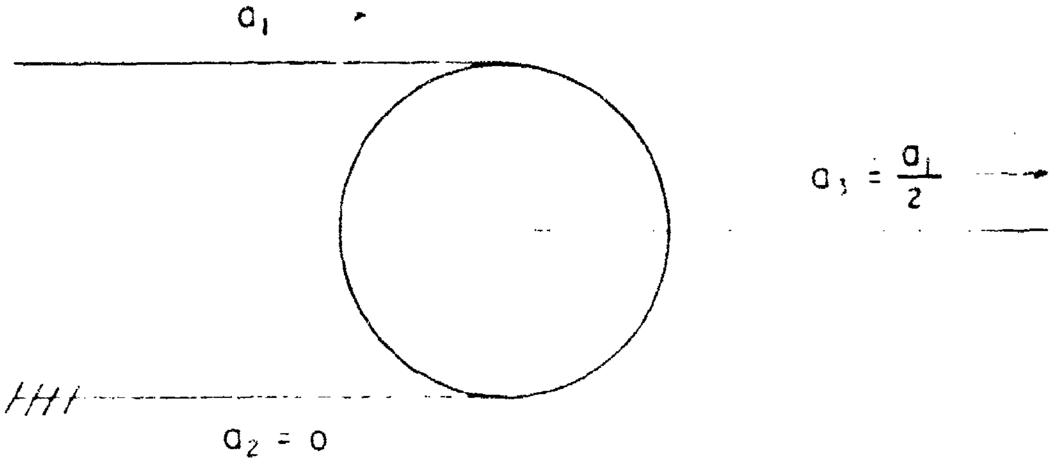
The quantity in brackets in equation (24) may be termed the total equivalent length. From equations (21) and (24),

$$\frac{F_3}{w} = 2\frac{F_2}{w} - \left[L_s + \pi r\right] a \quad . \tag{25}$$

Typical values of L_s are shown in the table on page 14. While the values for the equivalent lengths of various sheaves vary over a considerable range, it is reasonable to use an equivalent length of 25 ft. for any sheave in the present calculations. It may be noted that the equivalent length in feet is approximately the same as the sheave to cable diameter ratio, D/d, for the sheaves considered.

<u>Moving Sheave with One Cable Dead-Ended</u>

In this case as shown in the adjacent sketch, $v_2 = 0$, $a_2 = 0$, $v_3 = v_1/2$, and $a_3 = a_1/2$.



SHEAVE CHARACTERISTICS

	Cable Data*			Sheave Data*					Computed	
Catapult	Dia., d, Inches	Type	Linear Wt., lbs/ft.	Dia., D, Inches	Type	Material	Weight, lbs.	Radius of Gyration, Inches	$\frac{D}{d}$	Equivalent Length, Feet
H-3	1-1/2	6 x 37	3.49	48	Fairlead	Alum.	263	16.9	32	37
	1-1/2	6 x 37	3.49	48	X-head	Alum.	230	17.05	32	33
H-4	1-5/8	6 x 37	4.09	48	Fairlead	Steel	247	16.2	29.5	28
	1-5/8	6 x 37	4.09	30	Fairlead	Alum.	129	10.5	18.5	15
	1-5/8	6 x 37	4.09	48	X-head	Alum.	221	16.8	29.5	26
C-10	1-3/4	6 x 19	4.9	38½	Fairlead	Alum.	256	12.9	22	24
	[illegible]	6 x 19	6.4	44	X-head	Alum.	329	14.6	22	23
	[illegible]	6 x 19	6.4	51	X-head	Alum.	387	16.8	25.5	26
XH-3	2-1/4	6 x 37	7.85	60	Fairlead	Alum.	539	13.2	26.7	25

* Data are from, "Catapult Cable and Sheave Characteristics," Test Data, 4ND-NAMC-2230A.

Then from equations (20), (21) and (23)

$$\frac{F_2}{W} - \frac{F_1}{W} = \left[\frac{L_s}{2} + \frac{\pi r}{2}\right] a_1 \tag{26}$$

and

$$\frac{F_3}{W} = 2\,\frac{F_2}{W} + \left[\frac{W_s}{2W} + \frac{W_T}{2W} - \frac{L_s}{2}\right] a_1 \quad . \tag{27}$$

It should be noted here that the total equivalent length as shown in brackets in equation (26) is only half that of equation (24). The reason is that the angular acceleration here is only half as great as that in the previous case.

Multiply Reeved Sheave Systems

Consider a system of moving and fixed sheaves of multiplication n. There are n/2 moving sheaves, and (n/2 - 1) fixed sheaves, or (n - 1) sheaves in all. The system is shown in Figure 3, page 16. Let the fixed sheaves be designated by i = 1, 2, 3, ... n/2 - 1, the moving sheaves by j = 1, 2, 3, ... n/2, and the cable segments by k = 1, 2, 3, ... n - 1, all in order from the dead end of the cable. The acceleration as indicated in the figure is zero in the first cable segment, a in the last segment, and 2i a/n in the two cable segments in contact with the fixed sheave i. The linear acceleration of the whole set of moving sheaves is a/n.

In subsequent analyses it is convenient to use a quantity here defined as the "effective length" of certain cables. Since most of the cables in a multiply reeved system have accelerations less than the maximum acceleration in the system, force calculations are simplified by considering the "effective length" of any cable as equal to the actual length multiplied by the ratio of the acceleration of that cable to the maximum acceleration in the system. If the center to center distance between the two sets of sheaves is Z ft., the effective length L_{ei} of the two cable segments in contact with the fixed sheave i is

$$L_{ei} = \frac{2\,i}{n}\,Z \quad . \tag{28}$$

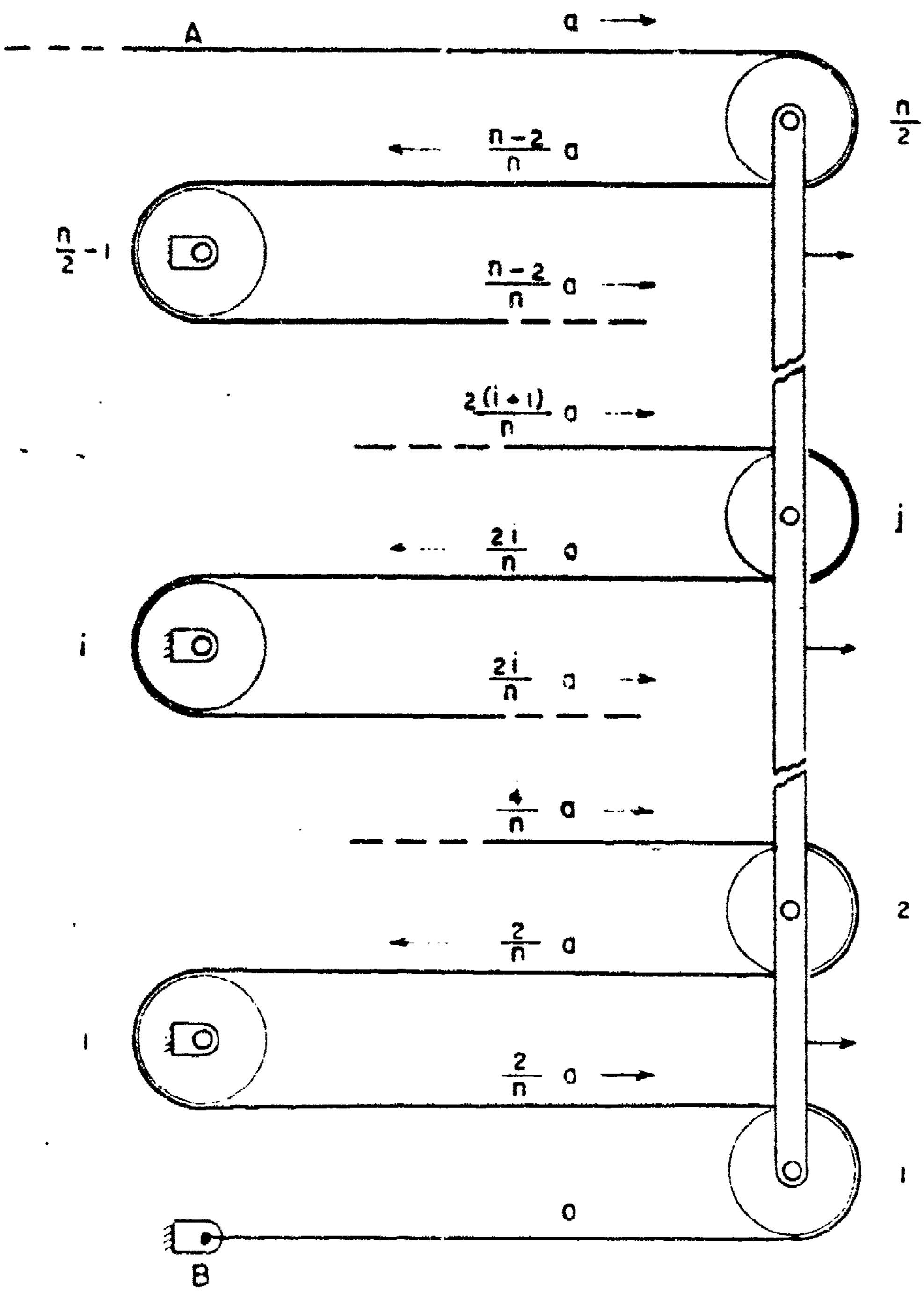


Figure 3. Multiply reeved sheave systems.

The total effective length of cable between points A and B in Figure 3 is L_c, given by

$$L_c = Z + 2 \sum_{i=1}^{n/2-1} \frac{2i}{n} Z = \frac{n}{2} Z \quad . \tag{29}$$

The cable segments in contact with the fixed sheave i are those designated 2i and 2i + 1. The difference in force between these two segments, from equation (22), is

$$\frac{F_{2i}}{w} - \frac{F_{2i+1}}{w} = \left[L_s + \pi r\right] \frac{2i}{n} a \quad , \tag{30}$$

in which L_s is the equivalent length of a single sheave. Thus L_{fi}, the effective length of fixed sheave i and the half turn of cable about it, is given by

$$L_{fi} = \frac{2i}{n} \left[L_s + \pi r\right] \quad , \tag{31}$$

and the total effective length of all the fixed sheaves and the cable wrapped on them, designated L_f, is given by

$$L_f = \sum_{i=1}^{n/2-1} \frac{2i}{n} \left[L_s + \pi r\right] = \frac{n-2}{4} \left[L_s + \pi r\right] \quad . \tag{32}$$

The cable segments in contact with the moving sheave j are those designated 2j - 1 and 2j. The difference in force between these two segments, from equation (20), is

$$\frac{F_{2j-1}}{w} - \frac{F_{2j}}{w} = \left[L_s + \pi r\right] \frac{1}{2} \left[\frac{2(j-1)}{n} a + \frac{2j}{n} a\right]$$

$$= \left[L_s + \pi r\right] \frac{2j-1}{n} a \quad . \tag{33}$$

This equation takes no account of forces required to produce a translational acceleration of the sheaves or of any possible crosshead; these forces are not transmitted by the cable in any practical catapult application. From equation (33), L_{mj}, the

total effective length of the moving sheave j and its half turn of cable, is

$$L_{mj} = \frac{2j - 1}{n}\left[L_s + \pi r\right] \quad . \tag{34}$$

and L_m, the total effective length of all the moving sheaves and the cable wrapped on them, is given by

$$L_m = \sum_{j=1}^{n/2} \frac{2j - 1}{n}\left[L_s + \pi r\right] = \frac{n}{4}\left[L_s + \pi r\right] \quad . \tag{35}$$

The total effective length L_t of the entire assembly between points A and B of Figure 3 is the sum of the individual effective lengths of cable segments and sheaves. Thus one finds from equations (29), (32) and (35)

$$L_t = \frac{n}{2} Z + \frac{n - 2}{4}\left[L_s + \pi r\right] + \frac{n}{4}\left[L_s + \pi r\right] \tag{36}$$

or

$$L_t = \frac{1}{2}\left[n Z + (n - 1) \pi r + (n - 1) L_s\right] \quad . \tag{37}$$

Since the total actual length of cable is $\left[n Z + (n - 1) \pi r\right]$, and there are $n - 1$ sheaves each of equivalent length L_s, the total effective length of the reeving system may be expressed in the form

Net effective length = $\frac{1}{2}$ x Sum of equivalent lengths of components.

Consider for example the system shown in Figure 4, drawn for a reeving system of multiplication $n = 4$. The length of run is assumed to be 200 ft.; there is assumed an additional 100 ft. of vertical cable. Since there are four sheaves exclusive of the reeving system, each of equivalent length 25 ft. as discussed earlier, the total effective length of the system, exclusive of the reeving, is 600 ft.

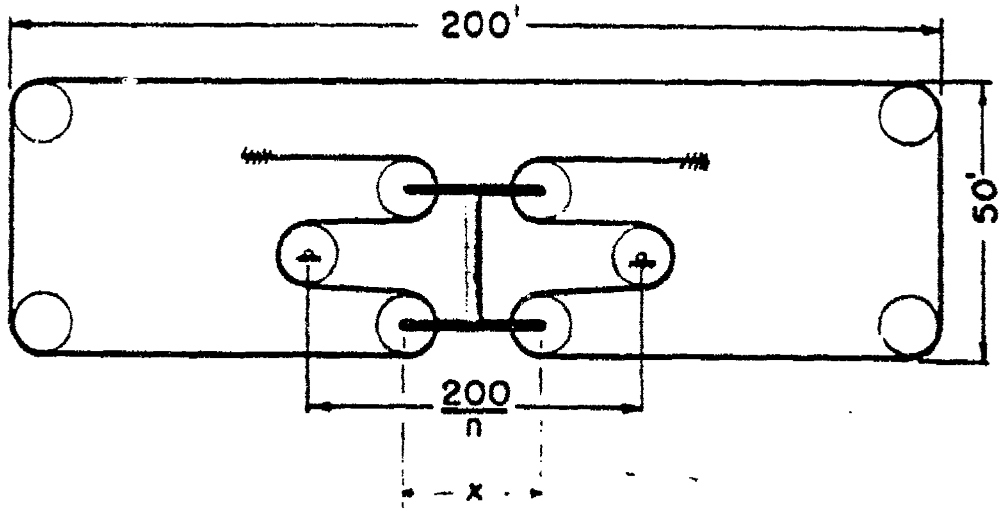


Figure 4. A multiply reeved cable system.

Neglecting the diameter of the sheaves and the distance x, one finds that with multiplication n, the minimum possible length of additional cable is

$$\frac{n - 1}{n} \times 200 \text{ ft.},$$

half of which will be effective; in all probability, the length will be somewhat greater than this. In addition there will be 2(n - 1) sheaves, each contributing (1/2) x 25 ft. to the effective length of the system. The net effect of these additions is as follows:

Multiplication	Length of Basic System	Additional Due to Reeving	Total Effective Length
1	600 ft.	0 ft.	600 ft.
2	600	75	675
4	600	150	750
6	600	208	808
8	600	263	863
10	600	315	915
12	600	367	967
14	600	418	1018
16	600	469	1069

It is of interest to note that in this example, when the multiplication is increased by a factor of 16, the total effective length is increased only be a factor of less than 2. On the other hand, the unavoidable increase in effective cable length is extremely significant at high capacities, particularly in case a long run is necessary to attain the end speed desired. Both the length of run and the maximum acceleration are in general determined by other factors. The sole advantage of multiple reeving therefore lies in decreasing the length and speed of the engine stroke. However, this must always be done at the expense of increasing the effective length of cable, and in cable-drive systems already near their ultimate capacity, the addition of multiple reeving becomes impractical or even impossible from the standpoint of the over-all weight of the installation.

INDIRECT CABLE-DRIVE CATAPULT WITH RETRIEVING CABLE BRAKING

The previous examples have been concerned with the basic relationships involved in cable systems in a simplified form; the systems have not conformed to what is actually used in a practical application. In order to determine the characteristics of a typical cable-drive system, consider the indirect-drive catapult shown in Figure 5.

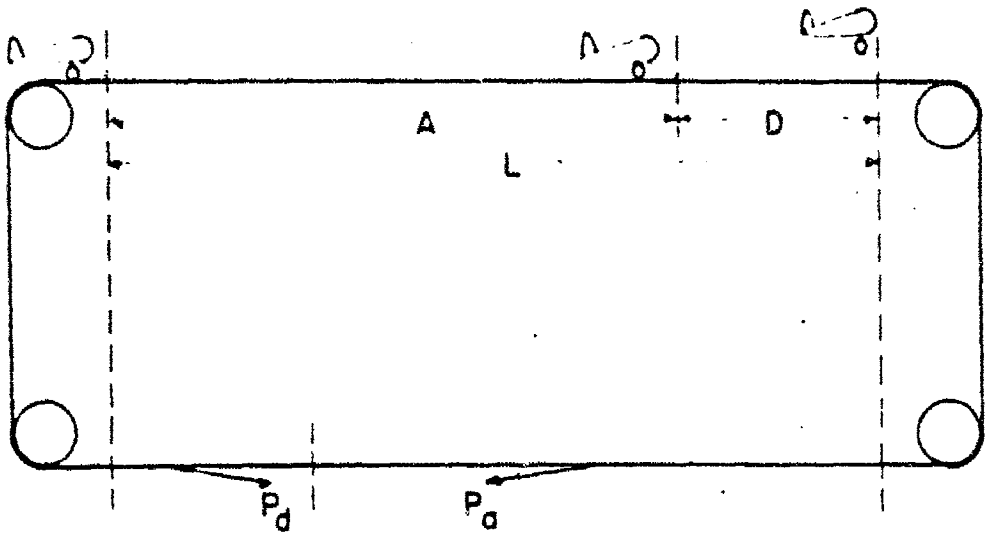


Figure 5. Indirect cable-drive catapult with retrieving cable braking.

One-to-One Reeving

Let it be assumed that the plane is accelerated for a length of run A to an end speed of v ft/sec., following which the cable and associated sheaves are decelerated in a length of run D to zero velocity. Let it further be assumed that the accelerating cable is of linear weight w_a lbs/ft. and is acted upon by the safe working load P_a lbs. Similarly, the decelerating and retrieving cable is of linear weight w_d lbs/ft. and is acted upon by its safe working load P_d lbs. The mass of the shuttle is neglected, and it is assumed that the cable does not transmit forces to change the kinetic energy of any part of the catapult engine. In the analysis the following notation will be used:

L = Total length of run (in ft.)

A = Length of accelerated run (in ft.)

D = Length of decelerated run (in ft.)

fL = Length of auxiliary cable required to connect the catapult to the engine, including the equivalent length of the sheaves required (in ft.)

W = Weight of plane (in lbs.)

P_a = Applied (safe working) load for driving cable (in lbs.)

w_a = Driving cable linear weight (in lbs/ft.)

P_d = Applied (safe working) load for retrieving cable (in lbs.)

w_d = Retrieving cable linear weight (in lbs/ft.)

a = Acceleration during accelerated run A (in g's.)

d = Deceleration during decelerated run D (in g's.)

y = w_d/w_a = P_d/P_a = Ratio of cross-sectional areas of retrieving and driving cables, assumed to be of same material and construction.

v = End speed at end of accelerated run (in ft/sec.)

E = Kinetic energy of plane at take-off (in ft-lbs.)

During the accelerated run, the plane and all of the cable are accelerated by the applied force P_a; application of Newton's Law yields:

$$P_a = \left[w_a \, (L + fL) + w_d \, (L + fL) + W \right] a \quad . \tag{38}$$

At the end of the accelerated run A, the velocity will be

$$v = \sqrt{2 \, a \, g \, A} = \sqrt{\frac{2 \, P_a \, g \, A}{w_a \, (L + fL) + w_d \, (L + fL) + W}} \quad . \tag{39}$$

Similarly for the decelerated run,

$$P_d = \left[w_a \, (L + fL) + w_d \, (L + fL) \right] d \tag{40}$$

and

$$v = \sqrt{2 \, d \, g \, D} = \sqrt{\frac{2 \, P_d \, g \, (L - A)}{w_a \, (L + fL) + w_d \, (L + fL)}} \quad . \tag{41}$$

Elimination of A from equations (39) and (41) yields

$$v = \sqrt{\frac{2\,(P_a/w_a)\,g}{(1 + P_a/P_d)\,(1 + w_d/w_a)\,(1 + f) + W/w_a L}} \tag{42}$$

or since $y = w_d/w_a = P_d/P_a$,

$$v = \sqrt{\frac{2(P_a/w_a)\,g}{(2 + y + 1/y)\,(1 + f) + W/w_a L}} \quad . \tag{43}$$

From equation (38) one obtains

$$\frac{W}{w_a} = \frac{1}{a}\,\frac{P_a}{w_a} - (1 + y)\,(1 + f)\,L \quad . \tag{44}$$

Substitution of this result in equation (43) yields

$$v = \sqrt{\frac{2\,(P_a/w_a)\,g}{(1 + \frac{1}{y})\,(1 + f) + (1/aL)\,(P_a/w_a)}} \quad . \tag{45}$$

In equations (44) and (45), P_a/w_a = 10,000 ft. as in previous cases, and g = 32.2 ft/sec^2.; hence

$$W = \frac{w_a}{a}\left[10{,}000 - (1 + y)\,(1 + f)\,(a\,L)\right] \tag{46}$$

and

$$v = \sqrt{\frac{644{,}000\,(aL)}{10{,}000 + (1 + 1/y)\,(1 + f)\,(aL)}} \quad . \tag{47}$$

The energy imparted to the plane at take-off is given by $E = Wv^2/2g$; thus from equations (46) and (47) one finds

$$E = \frac{10{,}000\,w_a}{a}\left[\frac{\left[10{,}000 - (1 + y)\,(1 + f)\,(aL)\right]\,(aL)}{10{,}000 + (1 + 1/y)\,(1 + f)\,(aL)}\right] \quad . \tag{48}$$

In applying these results to a typical catapult installation, it is reasonable to assume $f = 0.50$; that is, that the total equivalent lengths of the tow cable and the retrieving cable are each 1.5 times the total length of run L. For further analysis it is also convenient to introduce two new variables W' and E' defined as follows:

$$W' = \frac{W\,a}{w_a} = 10{,}000 - 1.5\,(1 + y)\,(aL) \tag{49}$$

and

$$E' = \frac{\mathcal{E}\,a}{10{,}000\,w_a} = (aL)\,\frac{10{,}000 - 1.5\,(1 + y)\,(aL)}{10{,}000 + 1.5\,(1 + 1/y)\,(aL)}\ . \tag{50}$$

Both W' and E' have the dimensions of length. With $f = 0.50$, equation (47) becomes

$$v = \sqrt{\frac{644{,}000\,(aL)}{10{,}000 + 1.5\,(1 + 1/y)\,(aL)}}\ . \tag{51}$$

These three equations now define W', E' and v as functions of y and (aL) only.

The ratio of accelerated run length to total length, A/L, is of importance in determining catapult performance and size requirements. From equations (39) and (51) this ratio is easily shown to be given by

$$\frac{A}{L} = \frac{10{,}000}{10{,}000 + 1.5\,(1 + 1/y)\,(aL)} \tag{52}$$

for the special case under consideration.

Another factor of extreme importance in determining catapult performance is the ideal efficiency e, here defined as the ratio of the energy imparted to the plane at take-off to the total energy delivered to the moving system. In the terms used above, one finds from equations (48) and (52)

$$e = \frac{E}{P_a\,A} = \frac{W'}{10{,}000}\ . \tag{53}$$

This result holds for $P_a/w_a = 10{,}000$ ft. and for any value of f.

Values of v, W', E', e and A/L have been computed for a range of y from 0 to 1.2 and for values of (aL) over the entire physically realizable range. These results are shown in Figures 6-9, pages 26-29; in each figure the uppermost curve corresponds to zero plane weight, and the dashed line to maximum energy output as shown below.

From these curves, it may be seen that the values of v, W', E', e and A/L depend mardedly upon y as well as upon aL. For maximum energy output, the optimum value of y for a given value of aL can be found by setting $(\partial E'/\partial y)_{aL} = 0$. The optimum value of y, y_m, is given by

$$y_m = \frac{6667 - (aL)}{6667 + (aL)} \quad . \tag{54}$$

Values of y_m for optimum energy output from this equation are as follows:

aL	y_m
0 ft.	1.000
1000	.739
2000	.538
3000	.379
4000	.250
5000	.143
6000	.053
6667	.000

These values define the dashed line on the four sets of curves. The exact position of the curve is determined by the assumed values of f (0.50) and P_a/w_a (10,000 ft.); similarly the maximum allowable value of aL is determined by these assumed values, and is equal to $(P_a/w_a)/(1 + f)$, or 6667 ft. in the present instance.

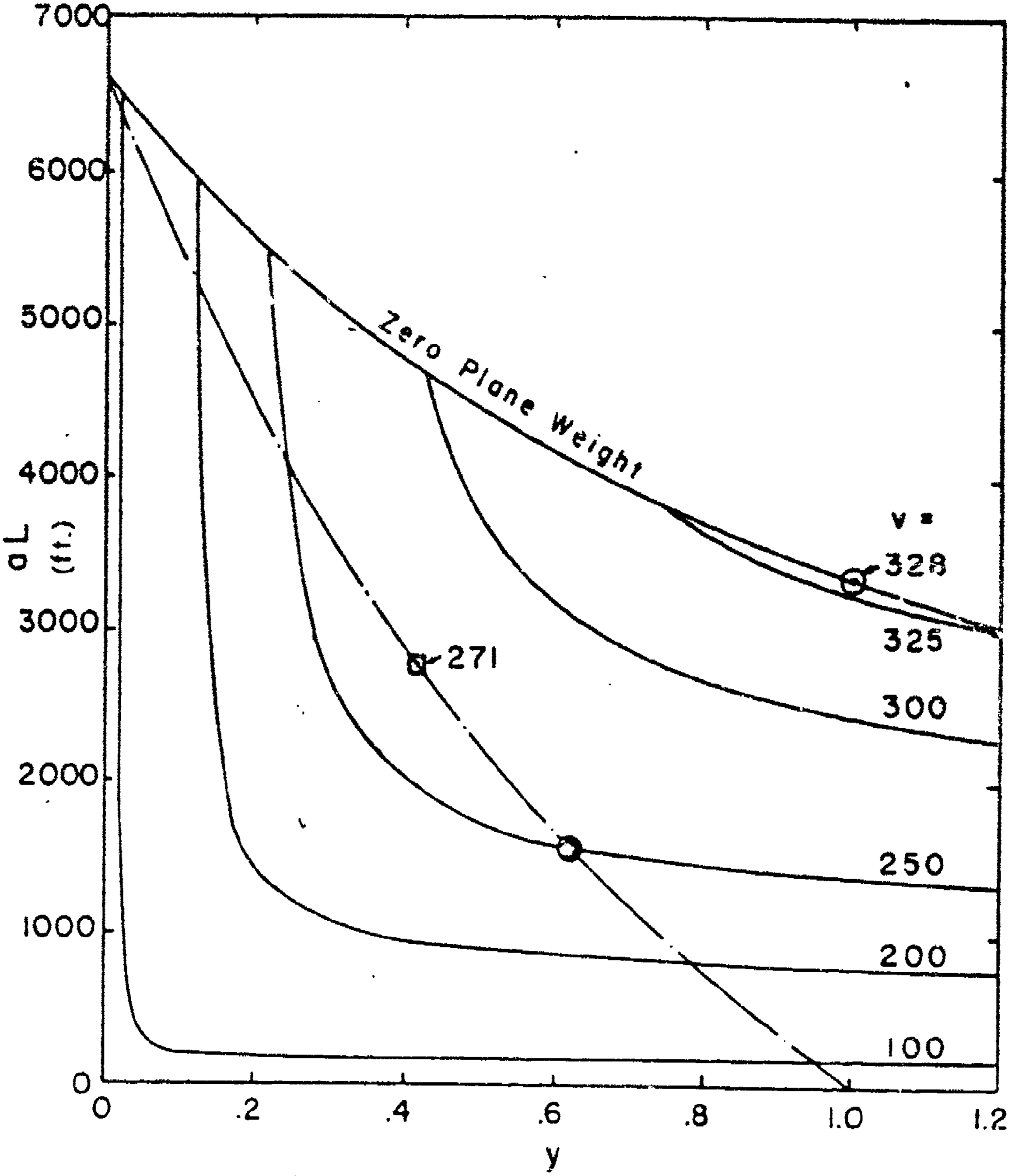


Figure 6. Values of v (in ft/sec.) as a function of aL and y for an indirect-drive catapult with retrieving cable braking.

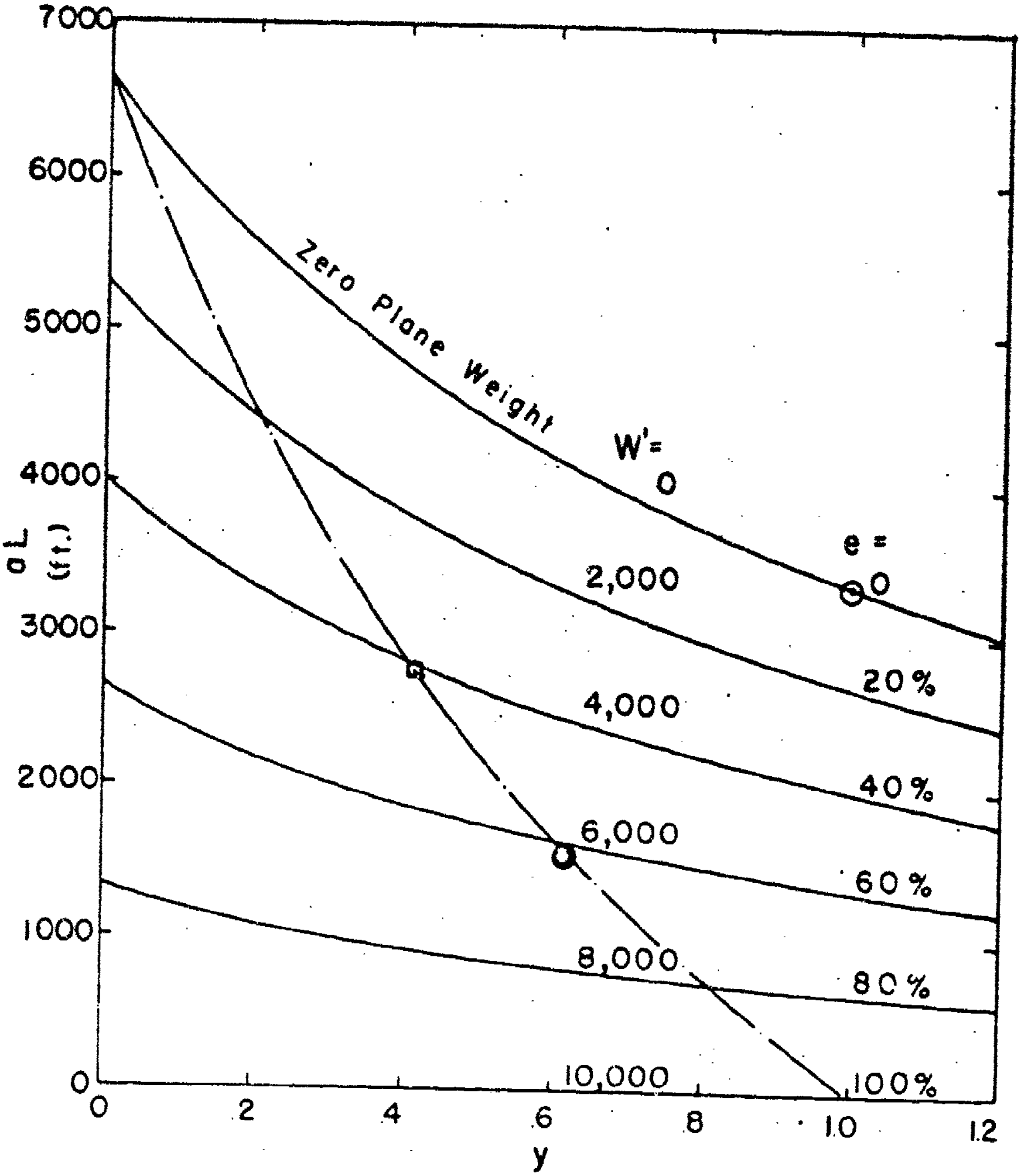


Figure 7. Values of W' (in ft.) and e (in per cent) as functions of aL and y for an indirect-drive catapult with retrieving cable braking.

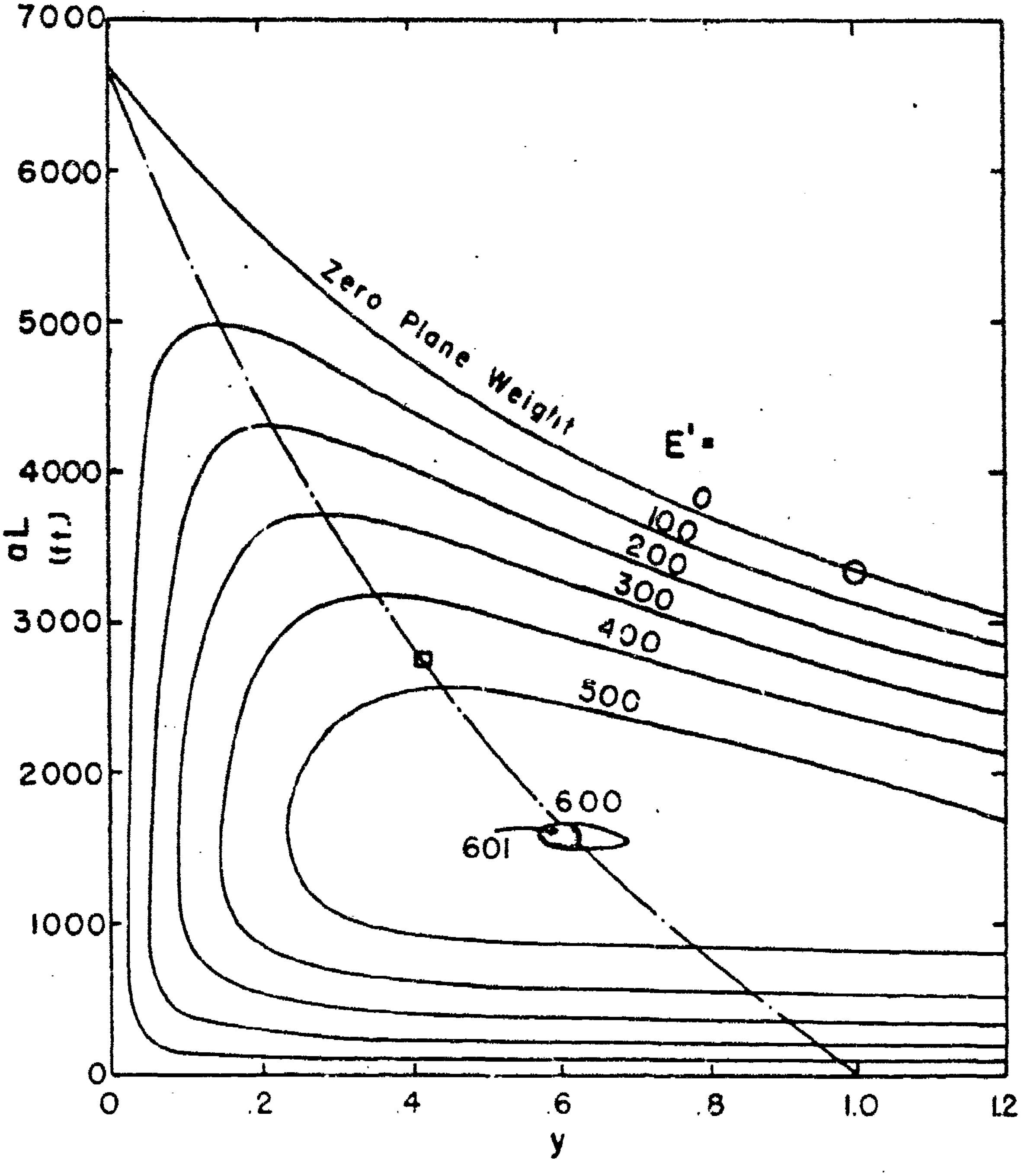


Figure 8. Values of E' (in ft.) as a function of aL and y for an indirect-drive catapult with retrieving cable braking.

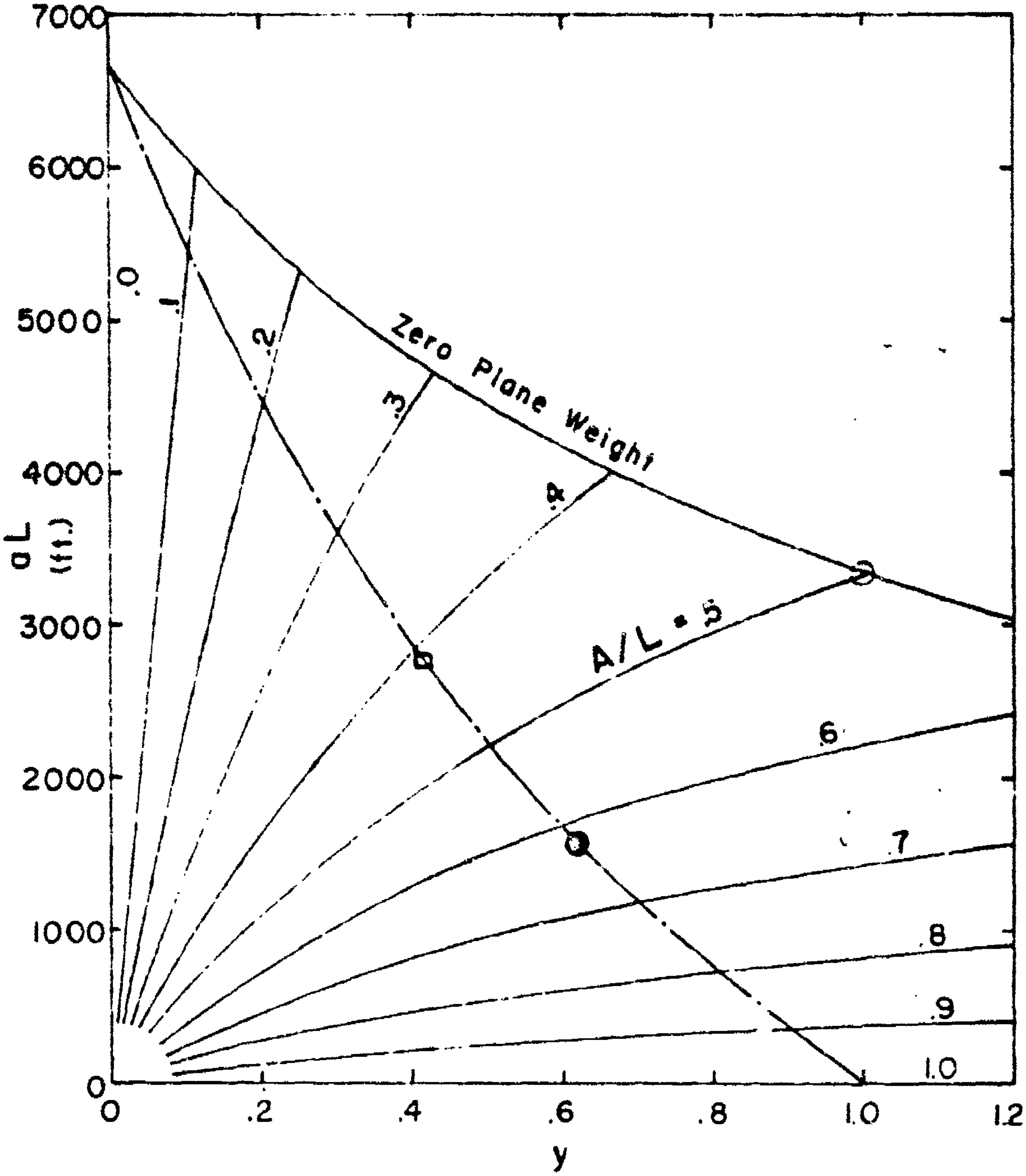


Figure 9. Values of A/L as a function of aL and y for an indirect-drive catapult with retrieving cable braking.

The maximum value of E' possible with this catapult may be found by setting

$$\left[\frac{\partial E'}{\partial (aL)}\right]_y = 0$$

and substituting equation (54) in the result. The coordinates of the maximum energy point and the energy at this point are

$$aL = 1574 \text{ ft.}$$

$$y = 0.618$$

$$E' = 601 \text{ ft.}$$

This point is shown as a solid circle on all four graphs.

From Figure 6, it is seen that the maximum velocity is obtained on the upper limiting line of performance (when $W' = 0$). The equation of this line is from (49),

$$aL = \frac{10,000}{1.5\ (1 + y)} \quad . \tag{55}$$

Substitution of this result in equation (51) yields $v°$, the velocity corresponding to zero plane weight, as a function of y.

$$v° = \sqrt{\frac{429,000}{2 + y + 1/y}} \tag{56}$$

The maximum value of $v°$ and the corresponding values of y and aL may be found by setting $dv°/dy = 0$ and solving for y. Thus one obtains

$$aL = 3333 \text{ ft.}$$

$$y = 1$$

$$v°_{max} = v_{max} = 328 \text{ ft/sec.} = 224 \text{ mph} = 194 \text{ knots.}$$

This point is shown on the graphs as an open circle.

The maximum velocity obtained on the maximum energy line (the dashed line in all figures), may be found from equations (51) and (54). Substitution of (54) in (51) yields v', the velocity for maximum energy output, as a function of aL.

$$v' = \sqrt{\frac{64.4\,(aL)\left[6667 - (aL)\right]}{6667 + (aL)}} \tag{57}$$

Setting $dv'/d(aL) = 0$ and solving for aL, y and v', one finds

$$aL = 2761 \text{ ft.}$$

$$\dot{y} = 0.414$$

$$v'_{max} = 271 \text{ ft/sec.} = 185 \text{ mph} = 160 \text{ knots.}$$

This point is shown in the graphs as an open square.

The curves in Figures 6-9 show that the possible take-off speed is definitely limited regardless of the size of d ving and retrieving cable used. The maximum energy for a gi.en size cable is also limited; it does not increase indefinitely as the length of run is increased.

The relationships among E', W' and v for increasing values of aL are shown in Figures 10 and 11, pages 32 and 33. Figure 10 shows the performance of a series of catapults in which the value of y is varied continuously so as to remain at the optimum value as aL is increased. With this condition, it may be noted from equations (49), (53) and (54) that the curves of y, e and $W' \times 10^{-4}$ as functions of aL all coincide. Figure 11 shows the performance with y constant at a value of 1.0 as aL is increased. Also plotted in Figures 10 and 11 are the values of end speed ($\sqrt{2\,a\,g\,L}$) to be obtained were the entire length of run used for acceleration.

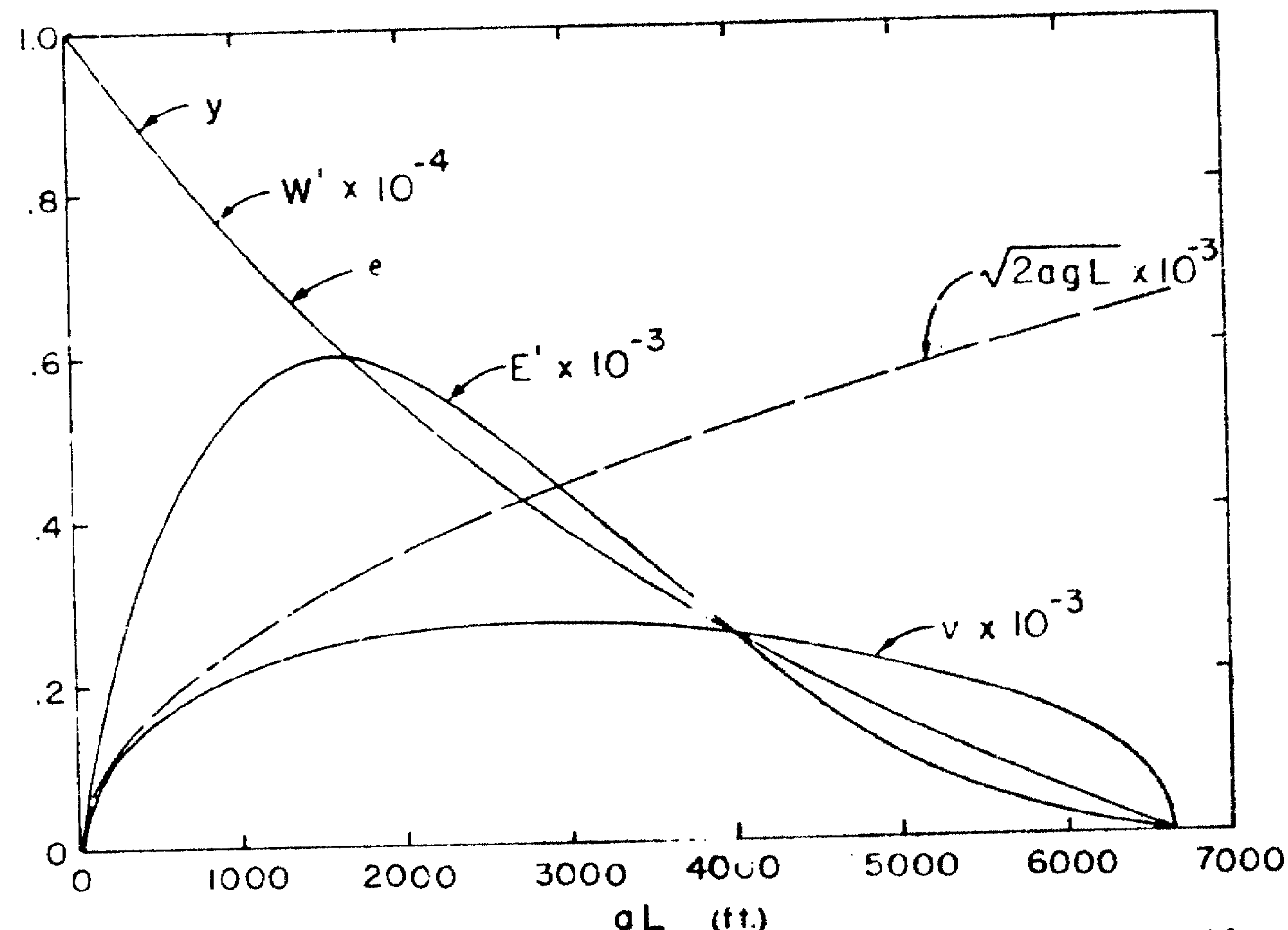


Figure 10. Performance of indirect-drive catapult with retrieving cable braking, shown with y varying so as to obtain maximum E' for each value of aL. Velocities are in ft/sec.; W' and E' are in ft.

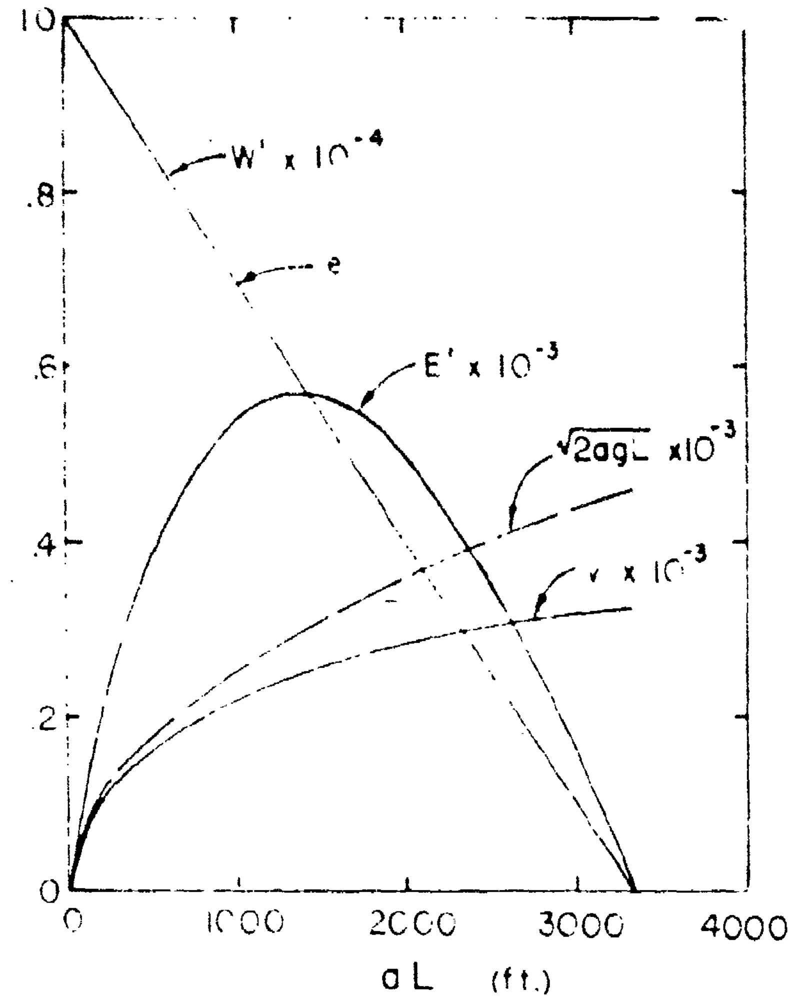


Figure 11. Performance of indirect-drive catapult with retrieving cable braking with y = 1. Velocities are in ft/sec.; W' and E' are in ft.

Multiple Reeving

Since the general aspects of indirect-drive cable systems with retrieving cable braking have been covered in the preceding section, the treatment of a multiply reeved system may be simplified to emphasize the actual effect of multiple reeving on speed limitations. The system is that shown in Figure 5 (page 21) modified to include multiple reeving of the sort indicated previously in Figure 4 (page 19). It is assumed that all cable has the same linear weight and maximum strength, that is, $y = 1$. In the notation previously used, the total equivalent length of cable, L_e, is given by

$$L_e = 2\,(1 + f)\,L + 4\,L_s + \left[\frac{n - 1}{n}\right]\frac{L}{2} + 2\,(n - 1)\,\frac{L_s}{2}\;, \qquad (58)$$

in which n is the multiplication and L_s the equivalent length of a sheave. This expression is easily obtained from the discussion on page 19. In addition to this effective length of cable, the piston and crosshead assembly must be accelerated and decelerated, but it is assumed that no cable-transmitted forces are involved. The weight of the shuttle, which generally will be relatively small, may be neglected in the approximate treatment.

If now the maximum load P is applied continuously during acceleration,

$$P = (L_e\,w + W)\,a\;, \qquad (59)$$

in which w is the cable linear weight, W the plane weight, and a the relative acceleration in units of g. At the end of the accelerated run of length A, the velocity v is given by

$$v = \sqrt{2\,a\,g\,A} = \sqrt{2\,g}\,\sqrt{\frac{P\,A}{L_e\,w + W}}\;. \qquad (60)$$

During the deceleration run, if the safe working load P is again applied to the cable,

$$P = L_e\,w\,d\;. \qquad (61)$$

Since the velocity is to be reduced to zero in the distance $L - A$,

$$v = \sqrt{2\,d\,g\,(L - A)} = \sqrt{2\,g}\,\sqrt{\frac{P\,(L - A)}{L_e\,w}}\;. \qquad (63)$$

If the velocity v is eliminated from equations (60) and (62), one obtains

$$\frac{A}{L} = \frac{L_e\ w + W}{2\ L_e\ w + W} \quad , \tag{63}$$

and the substitution of this result in equation (60) yields for the velocity v,

$$v = \sqrt{2\ g}\ \sqrt{\frac{P\ L}{2\ L_e\ w + W}} \quad . \tag{64}$$

Since L_e increases with increasing multiplication, it is obvious that the maximum velocity v must decrease with increasing multiplication. For specific illustration the following values of the various quantities may be assumed consistent with calculations made in the several other sections of this report;

$P =$ 100,000 lbs. $\quad$ $w =$ 10 lbs/ft.

$L =$ 200 ft. $\quad$ $L_s =$ 25 ft.

Let us take $f = 0.25$, the value appropriate to the system shown in Figure 4. Substitution of these values in equations (58) and (64) yields

$$L_e = 600 + 100 \left[\frac{n - 1}{n}\right] + 25\ (n - 1)\ \text{ft.} \tag{65}$$

and

$$v = \frac{35900}{\sqrt{W + 12000 + 500\ (n - 1) + 2000\ (n - 1)/n}}\ \text{ft/sec.} \tag{66}$$

Since there is almost certainly a maximum acceptable acceleration, if the safe working load P is applied to the tow cable, then the plane weight W must exceed a certain minimum determined by the maximum acceleration through equation (59). If for example the maximum acceleration is 4g, then for $n = 1$ the weight W must exceed 19000 lbs.; for higher values of n, lower corresponding limits may be calculated for W. Obviously, lighter loads may be catapulted at a safe acceleration by applying less than the maximum working load P to the tow cable. It may be shown, however, that the limiting velocity in such case is exactly the same as for the heavier load if the maximum braking force is to be applied in each case. If the towing and

braking forces are to be equal for a light load, and less than the maximum allowable, then the limiting velocity for a light load is less than that attainable with the heavier load at the same acceleration (over a longer accelerating run) with the maximum load P applied to the cable.

Values of the limiting velocity v in ft/sec. calculated for selected values of the plane weight W and multiplication n are shown in the following table.

W in lbs.	For a Multiplication n					
	1	2	4	8	12	16
19,000	204	199	195	189	183	179
25,000	187	183	180	175	170	167
30,000	175	172	169	165	161	158
40,000	157	155	153	150	147	145
50,000	144	142	140	138	136	134

An alternative way of viewing the effect of multiple reeving is to consider the velocity as limited by the acceleration maximum mentioned earlier. If the acceleration must not exceed 4g, then one finds from equation (66)

$$W = \left[\frac{35900}{v}\right]^2 - 12000 - 500\,(n - 1) - 2000\left[\frac{n-1}{n}\right], \qquad (67)$$

but from the restriction $a \leqq 4$ and equation (59)

$$W \geqq 19000 - 250\,(n - 1) - 1000\left[\frac{n-1}{n}\right]. \qquad (69)$$

Elimination of W from these two relations yields the following expression for v', the upper limit on the velocity consistent with a maximum acceleration of 4g.

$$v' \leqq \frac{35900}{\sqrt{31000 + 250\,(n - 1) + 1000\,(n - 1)/n}} \text{ ft/sec.} \qquad (69)$$

Values of the limiting velocity v' and the corresponding values of minimum* plane weight W are given in the following table for selected values of the multiplication n.

n	1	2	4	8	12	16
v' ft/sec.	204	201	199	196	193	190
W lbs.	19,000	18,300	17,500	16,400	15,300	14,300

The ideal efficiency of a catapult installation depends on the degree of reeving, and is easily shown to decrease with increasing multiplication in the reeving. With the safe working cable load P and the length of run L fixed as above, and with the same values again assumed for the several other quantities, the ideal efficiency e is given by

$$e = \frac{W\ v^2/2\ g}{P\ A} = \frac{W}{L_0\ w + W} \quad . \qquad (70)$$

For the case considered,

$$e = \frac{W}{W + 6000 + 250\ (n - 1) + 1000\ (n - 1)/n} \quad . \qquad (71)$$

The table below lists values of the efficiency e in per cent for selected values of the plane weight W in lbs. and the multiplication n.

W in lbs.	For a Multiplication n					
	1	2	4	8	12	16
19,000	76.0	73.8	71.7	68.8	66.3	64.0
25,000	80.6	78.7	76.9	74.3	72.1	70.1
30,000	83.3	81.6	80.0	77.7	75.6	73.7
40,000	87.0	85.6	84.2	82.3	80.5	78.9
50,000	89.3	88.1	87.0	85.3	83.8	82.4

- -

* See last paragraph on page 35 for significance of "minimum."

Although the undesirable effect of increasing multiplication on the efficiency becomes less pronounced as the plane weight increases, it is nevertheless an important factor and definitely militates against the use of multiple reeving in high capacity indirect-drive catapults.

It should be recognized that concomitant with the decreases in maximum velocity and ideal efficiency described above, there must always be an increase in the total weight of an indirect installation as the degree of reeving is increased. With increased multiplication, the added cable, sheaves and yokes all increase the total weight. The contribution of the cable is given by $L\,w\,(n - 1)/n$, where L is the length of the entire run and w is the weight per foot of cable. The increased weight due to the sheaves is directly proportional to the number of sheaves in the reeving, and hence may be written as a function of the multiplication n. Thus

$$W_s = \text{Constant} \times (n - 1) \quad . \tag{72}$$

The crosshead must also increase in weight with increasing multiplication, probably almost linearly. As an approximation one may write for the weight W_c of the crosshead

$$W_c = \text{Constant} + \text{Constant} \times (n - 1) \quad . \tag{73}$$

A little consideration indicates that none of the constants in equations (72) and (73) is necessarily small, and the total increase in weight with increasing degree of reeving may be quite considerable. Thus maximum velocity, efficiency and total weight are all affected in an undesirable manner by increases in the degree of reeving.

INDIRECT CABLE-DRIVE CATAPULT WITH SHUTTLE BRAKING

The preceding section (page 20) has shown the generalized performance to be expected from a conventional indirect-drive catapult with a retrieving cable used for braking. It was seen that the retrieving cable size is of considerable importance in the performance, and the retrieving cable is itself an appreciable part of the accelerated mass causing the limitations in speed.

In the following analysis it is assumed that the retrieving cable is eliminated entirely, with braking accomplished at the shuttle after the end of the accelerated run. All other factors affecting performance are left unchanged; that is, the effective length of the driving cable is assumed to be 1.5 times the length of the total run, and the shuttle weight is neglected. The system considered is shown in Figure 12. Let the notation be the same as in the previous section, except that $P_a = P$, and $w_a = w$.

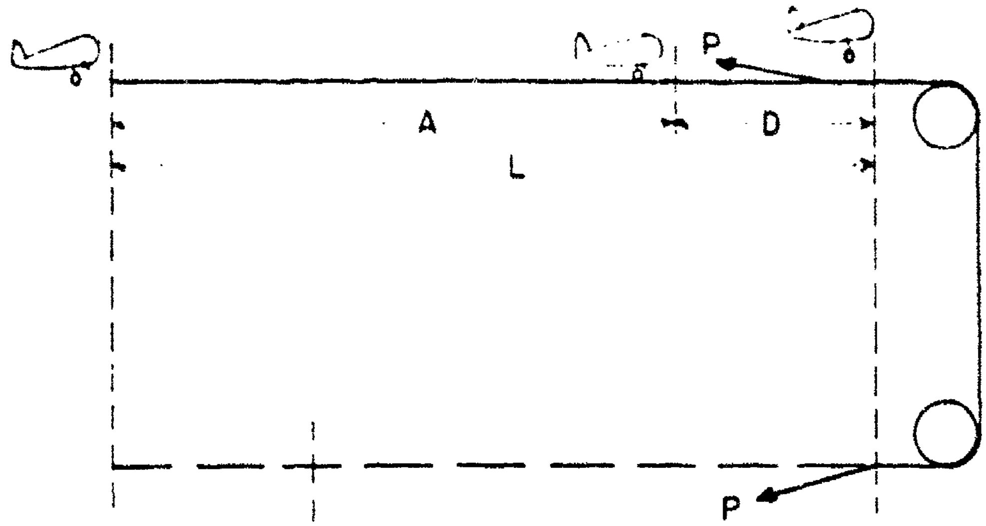


Figure 12. Indirect cable-drive catapult with shuttle braking.

During acceleration,

$$P = (1.5\ w\ L + W)\ a \tag{74}$$

and at the end of the accelerated run, the velocity is given by

$$v = \sqrt{2\ a\ g\ A} = \sqrt{\frac{2\ P\ g\ A}{1.5\ w\ L + W}}\ . \tag{75}$$

Similarly for the decelerated run,

$$P = 1.5\ w\ L\ d \tag{76}$$

and

$$v = \sqrt{2\ d\ g\ D} = \sqrt{\frac{2\ P\ g\ (L - A)}{1.5\ w\ L}}\ . \tag{77}$$

Eliminating A from equations (75) and (77) one finds

$$v = \sqrt{\frac{2\ P\ g\ L}{3\ w\ L + W}} \quad . \tag{78}$$

From equation (74) the plane weight W is given by

$$W = \frac{P}{a} - 1.5\ w\ L \quad , \tag{79}$$

which may be substituted in equation (78) to yield

$$v = \sqrt{\frac{2\ (P/w)\ g\ (aL)}{(P/w) + 1.5\ (aL)}} \quad . \tag{80}$$

With $P/w = 10{,}000$ ft. and $g = 32.2$ ft/sec^2., equations (79) and (80) become

$$W' = \frac{Wa}{w} = 10{,}000 - 1.5\ (aL) \tag{81}$$

and

$$v = \sqrt{\frac{644{,}000\ (aL)}{10{,}000 + 1.5\ (aL)}} \quad . \tag{82}$$

The energy imparted to the plane at take-off is given by

$$E = \frac{W\ v^2}{2\ g} \tag{83}$$

and from equations (81) and (82) one obtains

$$E = 10{,}000\ (aL)\ \frac{w}{a}\ \frac{10{,}000 - 1.5\ (aL)}{10{,}000 + 1.5\ (aL)} \tag{84}$$

or

$$E' = \frac{E\ a}{10{,}000\ w} = (aL)\ \frac{10{,}000 - 1.5\ (aL)}{10{,}000 + 1.5\ (aL)} \quad . \tag{85}$$

The ideal efficiency, e, defined as the ratio of energy imparted to the plane to the total imparted to the plane plus the moving cable, is given by

$$e = \frac{W}{W + 1.5\ w\ L} \quad . \tag{86}$$

Thus from equation (81) one obtains for e the same result as in the previous section,

$$e = \frac{W'}{10,000} \quad . \tag{87}$$

The ratio A/L is found from equations (75), (78) and (81) to be given by

$$\frac{A}{L} = \frac{10,000}{10,000 + 1.5\ (aL)} \quad . \tag{88}$$

The maximum value of E' and the corresponding value of aL may be found by setting $dE'/d(aL) = 0$; the values are

$$E'_{max} = 1144 \text{ ft.}$$

$$aL = 2761 \text{ ft.}$$

The maximum value of v occurs for aL = 6667 ft. (i.e., when W = 0), and is found from equation (82) to be

$$v_{max} = 463 \text{ ft/sec.} = 316 \text{ mph} = 274 \text{ knots.}$$

W', E' and v are shown as functions of aL in Figure 13, page 42. Also included is a curve of $\sqrt{2\ g\ a\ L}$, the velocity attainable if the entire length of run is used for acceleration.

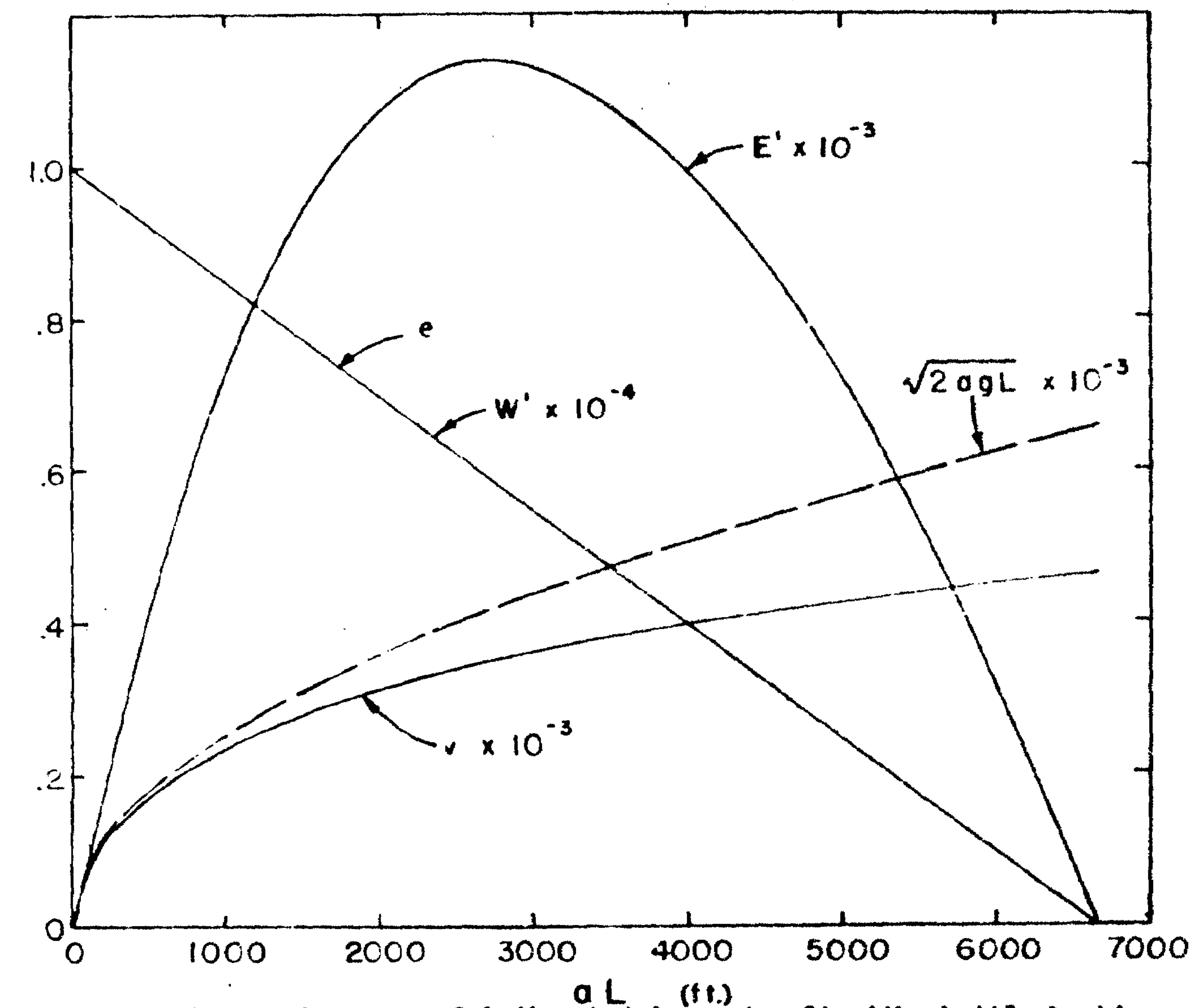


Figure 13. Performance of indirect-drive catapult with shuttle braking. Velocities are in ft/sec.; W' and E' are in ft.

INDIRECT CABLE-DRIVE CATAPULT WITH MULTIPLE BRAKING

A study of the preceding two sections (pages 20 and 38) shows that the capacity and speed of an indirect-drive catapult are limited by two factors:

1. The energy that is absorbed in accelerating the cable and is therefore not available for the acceleration of the plane.

2. The distance that is required for the deceleration of the cable.

The first of these two limiting factors is inherent in a cable-drive system and can be reduced only by using a cable material of higher strength to linear weight ratio, or, of course, by decreasing the effective length of cable required for a given total run.

The second factor, however, might be eliminated by one of several conceivable braking arrangements. For example, braking forces could be applied to the cable simultaneously at a number of points along its length so that any one brake would need decelerate only a small portion of the cable; or the shuttle could be detached from the cable at the end of the run for independent high-g braking, and the cable braking system so arranged that the cable could continue to move for some distance in a direction other than in the line of the accelerated run. With either system an independent light retrieving system must be provided. While it must be conceded that these schemes present serious practical difficulties, neither is impossible. In either case, the limiting performance would be that obtained with acceleration over the entire length of run and braking over a negligible distance.

In the following analysis of a catapult with either type of braking it is assumed, therefore, that the total length of run is used for acceleration, and that other factors remain the same as in the previous two analyses. During acceleration with the safe working load applied to the cable,

$$P = (1.5\, w\, L + W)\, a \tag{89}$$

or

$$W = \frac{P}{a} - 1.5\, w\, L \quad . \tag{90}$$

Thus at the end of the run the velocity v is given by

$$v = \sqrt{2 \, a \, g \, L} = \sqrt{\frac{2 \, P \, g \, L}{1.5 \, w \, L + W}} \quad . \tag{91}$$

The energy imparted to the plane is

$$E = \frac{W \, v^2}{2 \, g} = \frac{(P/a - 1.5 \, w \, L)\,(2 \, a \, g \, L)}{2 \, g} \quad . \tag{92}$$

Taking $P/w = 10,000$ ft. and $g = 32.2$ ft/sec^2., one finds

$$W' = \frac{Wa}{w} = 10,000 - 1.5 \, (aL) \tag{93}$$

$$v = \sqrt{64.4 \, (aL)} \tag{94}$$

$$E' = \frac{E \cdot a}{10,000 \, w} = \frac{(aL)}{10,000} \left[10,000 - 1.5 \, (aL) \right] \quad . \tag{95}$$

The ideal efficiency is, as before,

$$e = \frac{W}{W + 1.5 \, w \, L} = \frac{W'}{10,000} \quad . \tag{96}$$

The maximum value of E' and the corresponding value of aL may be found by setting $dE'/d(aL) = 0$; the values are

$$E'_{max} = 1667 \text{ ft.}$$

$$aL = 3333 \text{ ft.}$$

The maximum velocity and its corresponding value of aL are

$$v_{max} = 655 \text{ ft/sec.} = 446 \text{ mph} = 388 \text{ knots}$$

$$aL = 6667 \text{ ft.}$$

Note that this maximum velocity is necessarily less than the absolute limiting velocity for a straight cable without auxiliary cable connecting it to an engine; this latter limiting velocity was earlier shown to be 802 ft/sec. W', E' and v are shown as functions of aL in Figure 14.

THE SHUTTLE CLUTCH CATAPULT

Another type of cable-drive catapult which has been proposed consists of a continuously moving cable, and a shuttle which can be clutched to and detached from the cable. The cable, driven by a flywheel, moves at a velocity somewhat greater than the desired take-off speed and, at the start of the accelerated run, the shuttle is clutched to this cable. Initially the slippage between the cable and the shuttle is large, but it decreases as the plane progresses. At the end of the run the shuttle is detached and braked, while the cable is accelerated back to its original speed in preparation for the next shot. Such a system has been considered in NAMC Report No. M-5030, entitled, "Evaluation of Flywheel Type Catapults." In this report, catapults using standard wire rope, flat wire rope, and steel ribbon were considered. The general conclusion was that the use of none of the three would result in a practical catapult.

This group agrees with the conclusion stated above, but would like to point out that the use of multiple flat ribbons would eliminate some of the difficulties described in the report. Consider, for example, the multiple ribbon system shown in Figure 15. In this system the flywheel is fitted with rollers for the purpose of increasing the normal force, and hence the frictional force, between the ribbons and the flywheel. This system eliminates the need for multiple wraps on the flywheel, as well as the need for great tension in the portion of the system not transmitting force to the plane. While there is in this portion of the ribbon system a moderate tension furnishing the centripetal force acting on the ribbons as they pass around the pulleys, this force is developed by the ribbon itself; the only purpose of the tensioning pulleys, therefore, is to provide a take-up for the slack.

There is no slippage between ribbons as they pass over the sheaves, but between sheaves slight slippage occurs because the linear velocities of adjacent ribbons are different. Such slippage is unimportant in determining performance, and may be eliminated if desired for service life considerations by slightly separating the ribbons in the portions between sheaves.

With a multiple plate clutch as shown, the normal forces between pairs of surfaces are identical; hence the distribution of towing force and heating effect among the ribbons tends toward uniformity. It is necessary, of course, to provide ample cooling of the clutch by evaporative or other means.

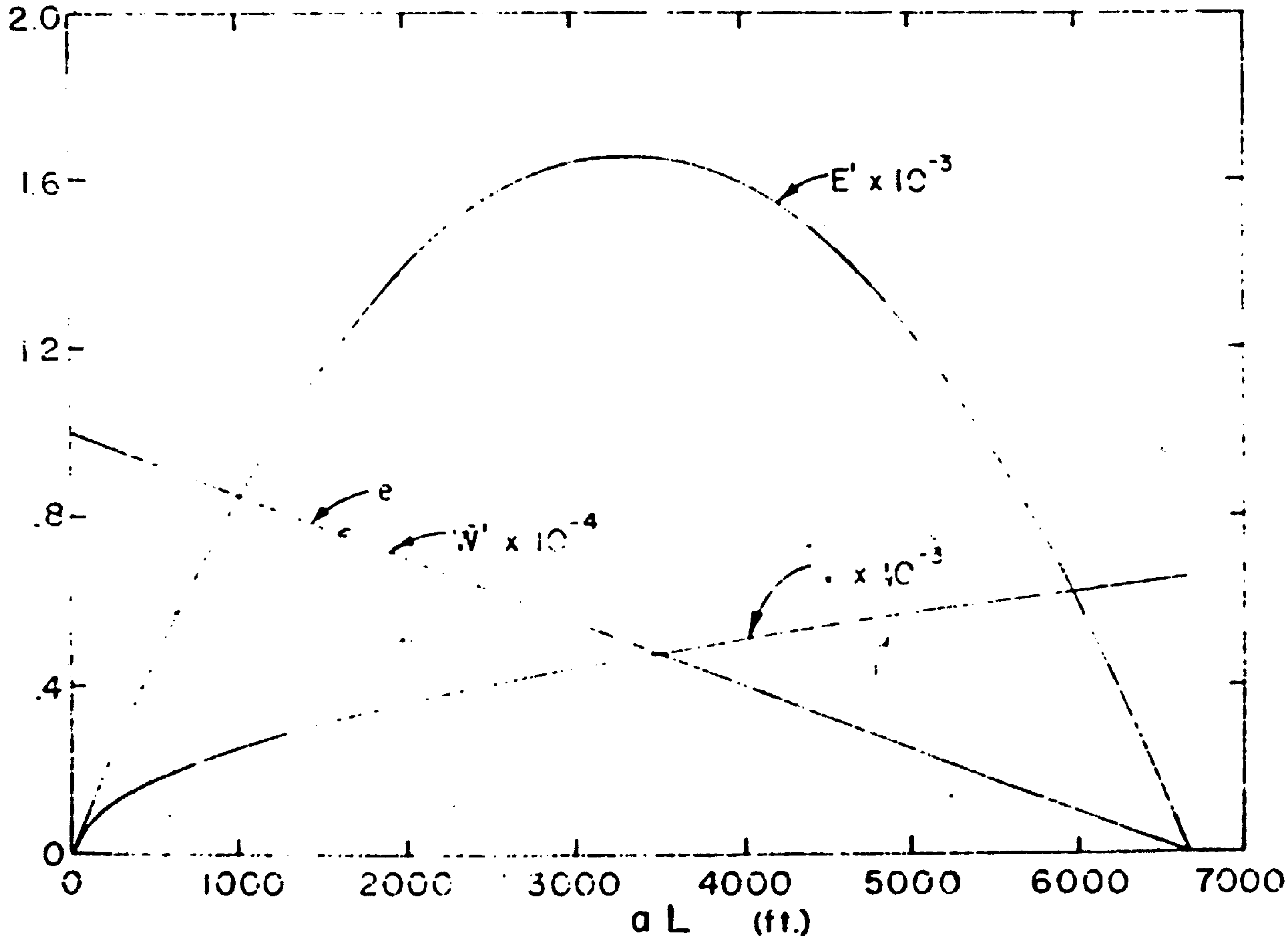


Figure 14. Performance of indirect cable-drive catapult with multiple braking. W' and E' are in ft.; v is in ft/sec.

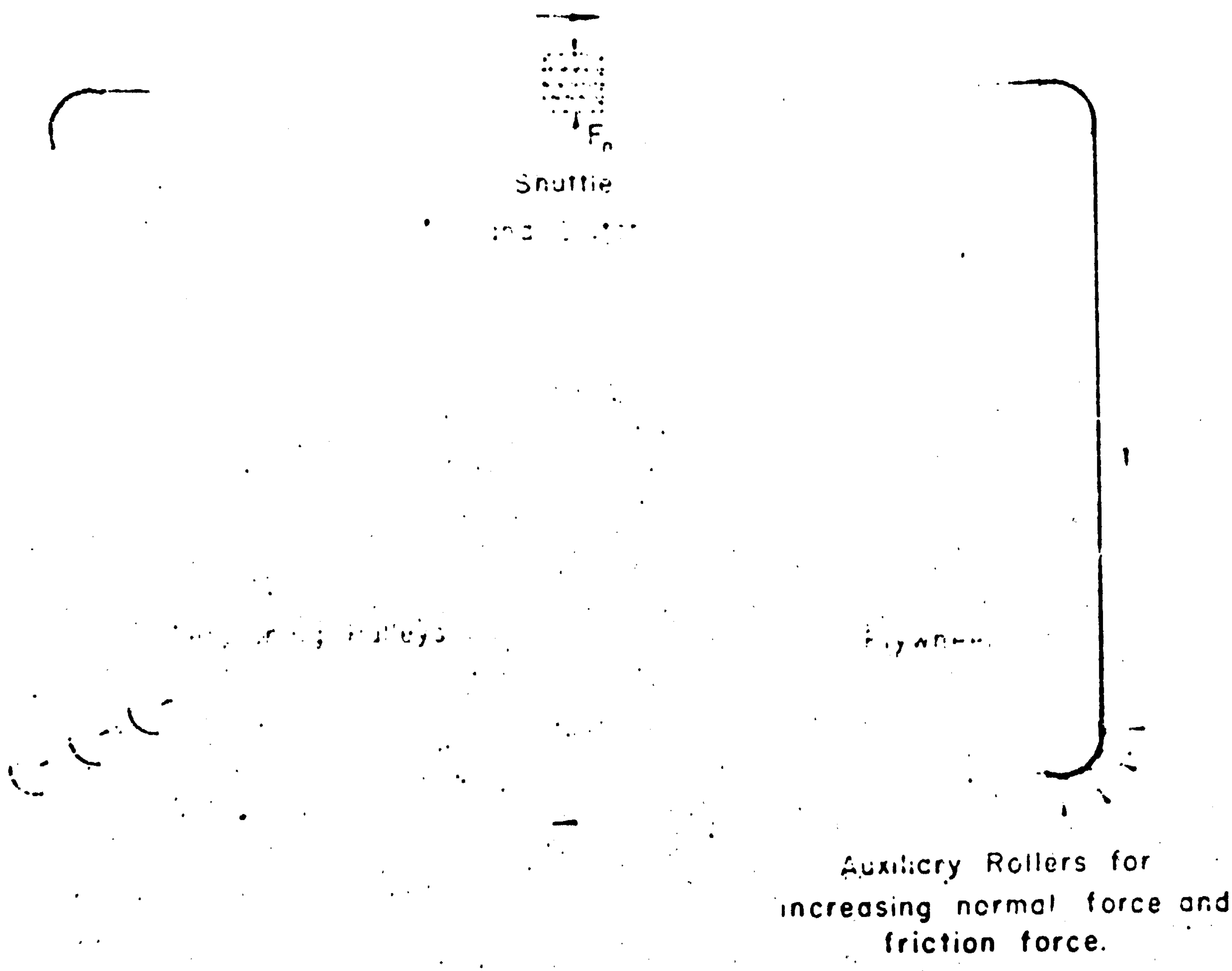


Figure 15. The shuttle clutch catapult.

It is realized that there are serious practical difficulties in constructing a system like the one suggested, particularly the clutching mechanism. However, it is not the purpose here to discuss the details of construction, but rather to consider the basic performance of such a system could it be made practical.

In order that the performance of an ideal shuttle clutch catapult can be compared with the performance of other catapults, let it be assumed that the towing ribbons have the characteristic strength to linear weight ratio of standard wire rope, and that the clutch is capable of developing the safe working load of the cable. Let it further be assumed that the braking of the shuttle takes place in a sufficiently short distance that it may be neglected.

Using the notation defined in previous sections, one finds for the accelerated run,

$$P = Wa \tag{97}$$

and

$$v = \sqrt{2\,a\,g\,L} \quad . \tag{98}$$

The energy imparted to the plane at take-off is given by

$$E = \frac{W\,v^2}{2\,g} = P\,L \quad . \tag{99}$$

With P/w again taken as 10,000 ft., and since $g = 32.2$ ft/sec^2., one obtains

$$W' = \frac{Wa}{w} = \frac{P}{w} = 10{,}000 \tag{100}$$

$$v = \sqrt{64.4\,(aL)} \tag{101}$$

$$E' = \frac{E\,a}{10{,}000\,w} = (aL) \quad . \tag{102}$$

The performance of this catapult is shown in Figure 16. It should be pointed out that this type of catapult is not an indirect-drive but rather a direct-drive type, and its performance is therefore identical with that of other direct-drive types of the same capacity.

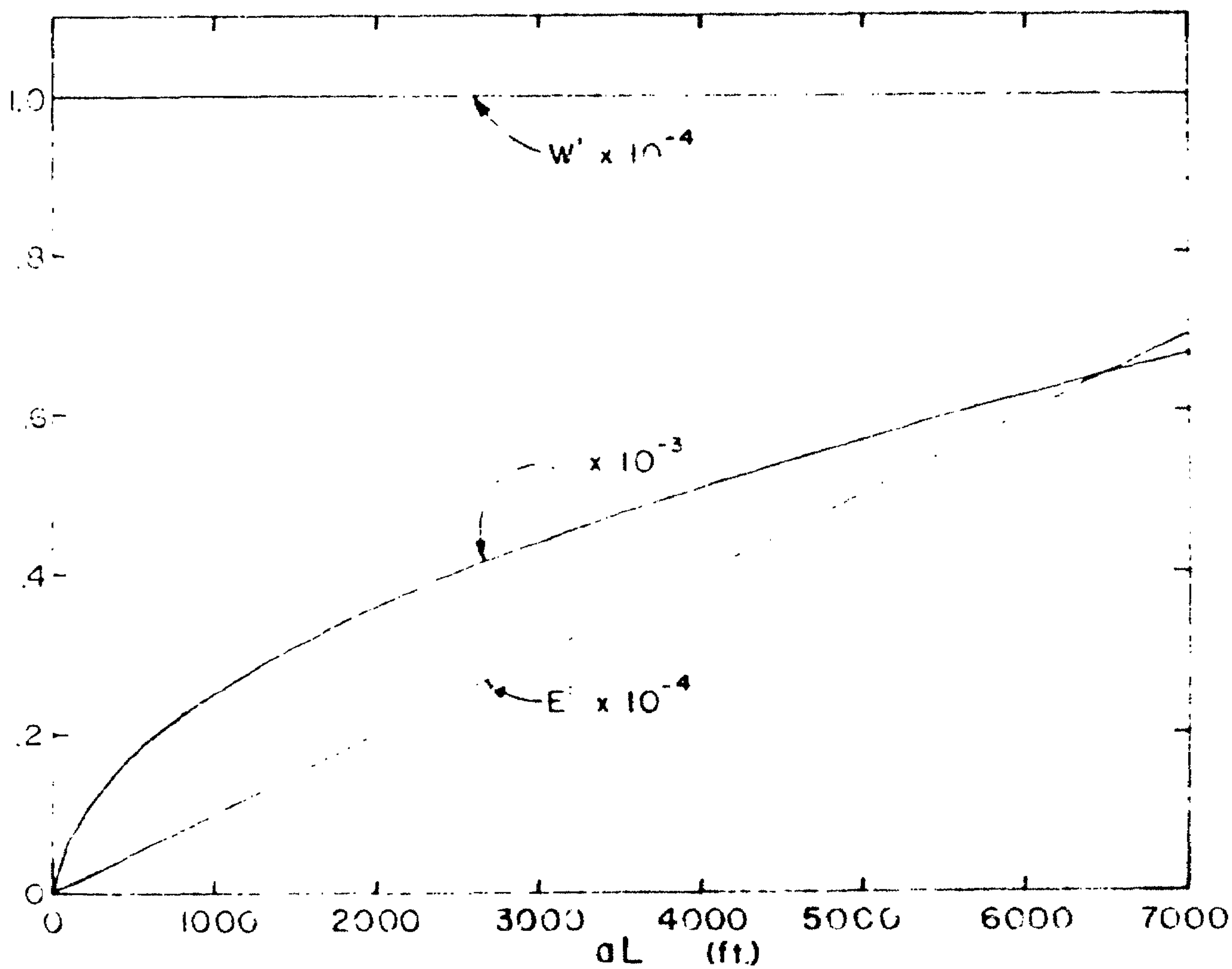


Figure 16. Performance of shuttle clutch catapult. E' and W' are in ft.; v is in ft/sec.

APPLICATION TO SPECIFIC CATAPULT SYSTEMS

In order to illustrate more clearly the relative performance of the four types of catapults of the preceding sections, the results of these sections will be applied to the discussion of specific examples. The lengths selected for these examples are 200 ft., 400 ft. and 800 ft.; the latter is included for possible application to land-based catapults. In all calculations it is assumed that the capacity of the engine is 100,000 lbs.; this value is also taken for the safe working load of the cable and corresponds approximately to that of a $2\frac{1}{2}$-inch cable weighing 10.0 lbs/ft. The results computed here may be applied directly to a catapult of the same length but having a cable weighing w lbs/ft. by the following equations where the starred values now represent those corresponding to the different linear weight w*:

$$W^* = W \frac{w^*}{10}$$

$$v^* = v$$

$$a^* = a$$

$$E^* = E \frac{w^*}{10}$$

There are four different cases corresponding to the four types of catapults discussed in the previous sections.

Case I: Indirect Cable-Drive Catapult with Retrieving Cable Braking (see page 20): If $y = 1$ and $f = 0.5$, then the relative acceleration a (the acceleration in units of g) is

$$a = \frac{100,000}{W + 30\ L} \quad .$$

The velocity v and energy output E are given respectively by

$$v = \sqrt{\frac{6,440,000\ L}{W + 60\ L}}$$

and

$$E = \frac{W\ v^2}{2\ g} \quad .$$

Case II: Indirect Cable-Drive Catapult with Shuttle Braking (see page 38): It is assumed that the shuttle and tow cable are braked by a braking force applied at the shuttle, and that no retrieving cable is used. The length of run used in the braking phase may be determined from equation (88). If $f = 0.5$, then

$$a = \frac{100{,}000}{W + 15\ L} \quad .$$

and the terminal velocity v and energy output E are given by

$$v = \sqrt{\frac{6{,}440{,}000\ L}{W + 30\ L}}$$

$$E = \frac{W\ v^2}{2\ g} \quad .$$

Case III: Indirect Cable-Drive Catapult with Multiple Braking (see page 43): It is assumed that some means of braking is provided, such as discussed on page 43, so that the entire length of run is available for acceleration. If $f = 0.5$ as before, then one finds for the relative acceleration a, terminal velocity v, and energy output E the following:

$$a = \frac{100{,}000}{W + 15\ L}$$

$$v = \sqrt{\frac{6{,}440{,}000\ L}{W + 15\ L}}$$

$$E = \frac{W\ v^2}{2\ g} \quad .$$

Case IV: The Shuttle Clutch Catapult, or other direct-drive catapults (see page 45): An independent means of braking the shuttle in a negligibly short distance must be provided at the end of the acceleration run. An ideal clutch is assumed, that is, one capable of developing the safe working load of the cable. The relative acceleration a, terminal velocity v, and energy output E are given respectively by

$$a = \frac{100{,}000}{W}$$

$$v = \sqrt{\frac{6{,}440{,}000\ L}{W}}$$

$$E = \frac{W\ v^2}{2\ g} = 100{,}000\ L \quad .$$

The relative acceleration a is shown in Figure 17 as a function of plane weight W for all four types of catapults of 200 ft. length. An arbitrary maximum acceleration of 4g is indicated in the figure. It is interesting to note that the acceleration attains a finite limit for zero plane weight. The terminal velocity v and energy output E for the four cases for catapult runs of 200 ft., 400 ft. and 800 ft. are shown in Figures 18, 19 and 20. The breaks in all the curves arise from the fact that in all cases the acceleration was limited to a maximum of 4g, as indicated in Figure 17 for catapults of 200 ft. length; the equations above were suitably modified to take this limitation into account.

In all of the curves it should be noted that the performance decreases in the order of Cases IV, III, II and I. The difference in performance is small in the 200 ft. catapult but becomes quite large in the 800 ft. catapult. For the greater length, the conventional catapult of Case I can not attain an acceleration of 4g except for a plane weight of 1,000 lbs. or less.

The variations in relative performance of these four types of catapults are further emphasized in Figures 21, 22 and 23, which show the velocity and energy output for Cases I, II and III as percentages of those for a direct-drive catapult (Case IV).

Figure 21 for a conventional catapult with retrieving cable braking (Case I) shows that the velocity and energy output are seriously reduced as the catapult length is increased. For the 200 ft. run the energy output averages about 75 per cent of that for the direct-drive; for the 400 ft. run it drops to about 60 per cent; and for an 800 ft. run it averages about 45 per cent with a minimum value of less than 35 per cent. It should be emphasized again that this energy decrease is due entirely to the decreased speed that can be obtained and not to a decrease in permissible weight; the weight is unlimited in both cases.

Figures 22 and 23 show similar results for Cases II and III, but the reduction in performance is much smaller than for Case I. Figure 23, for example, shows that with a 400 ft. run, the terminal velocity in Case III is only slightly less than that for the direct-drive catapult; and even with an 800 ft. run, the velocity over most of the range is more than 85 per cent of that for the direct-drive catapult, and the energy output over the weight range considered averages more than 80 per cent of that from a direct-drive catapult of the same length of run and engine capacity.

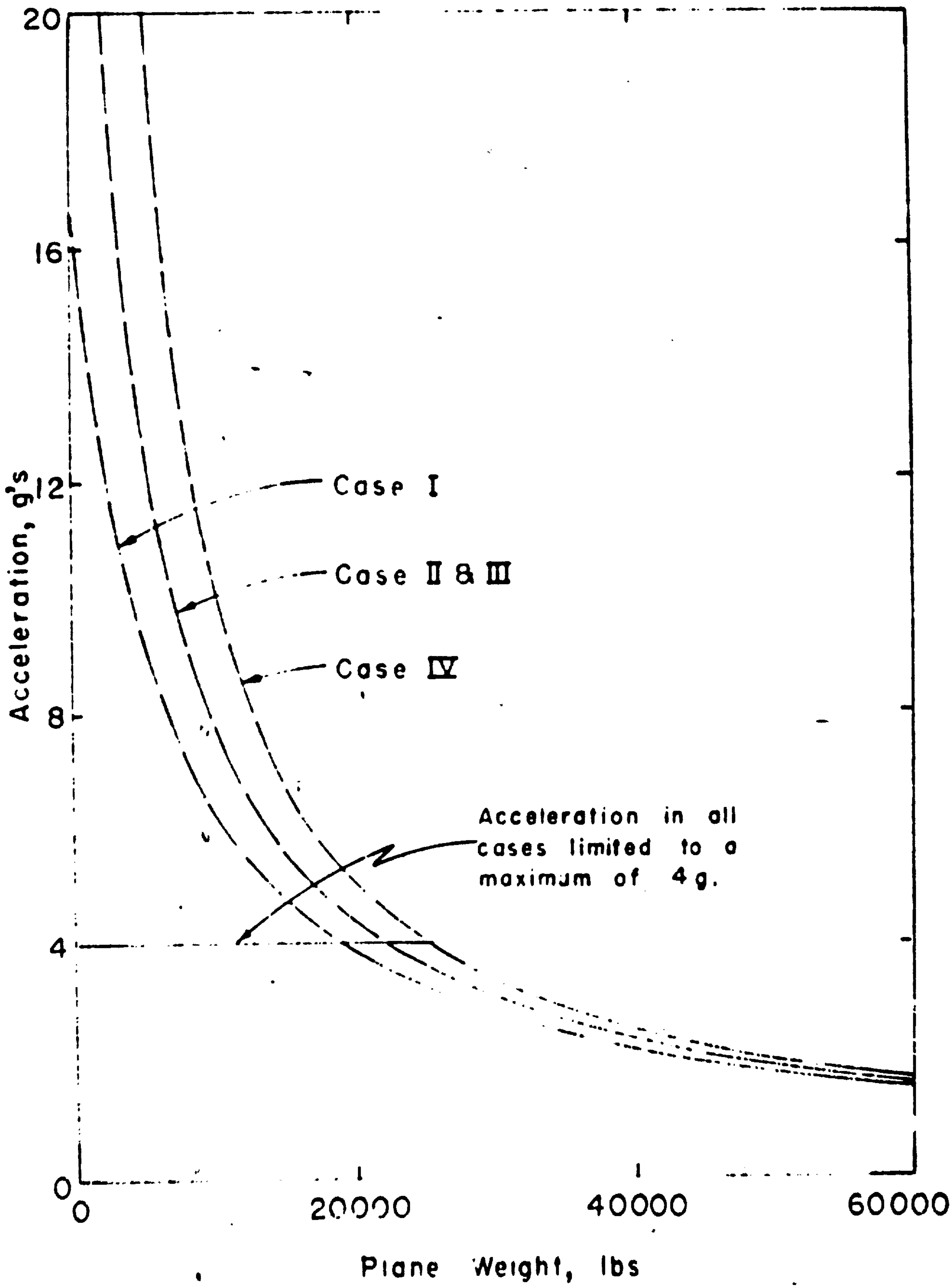


Figure 17. Acceleration for 200 ft. catapults.

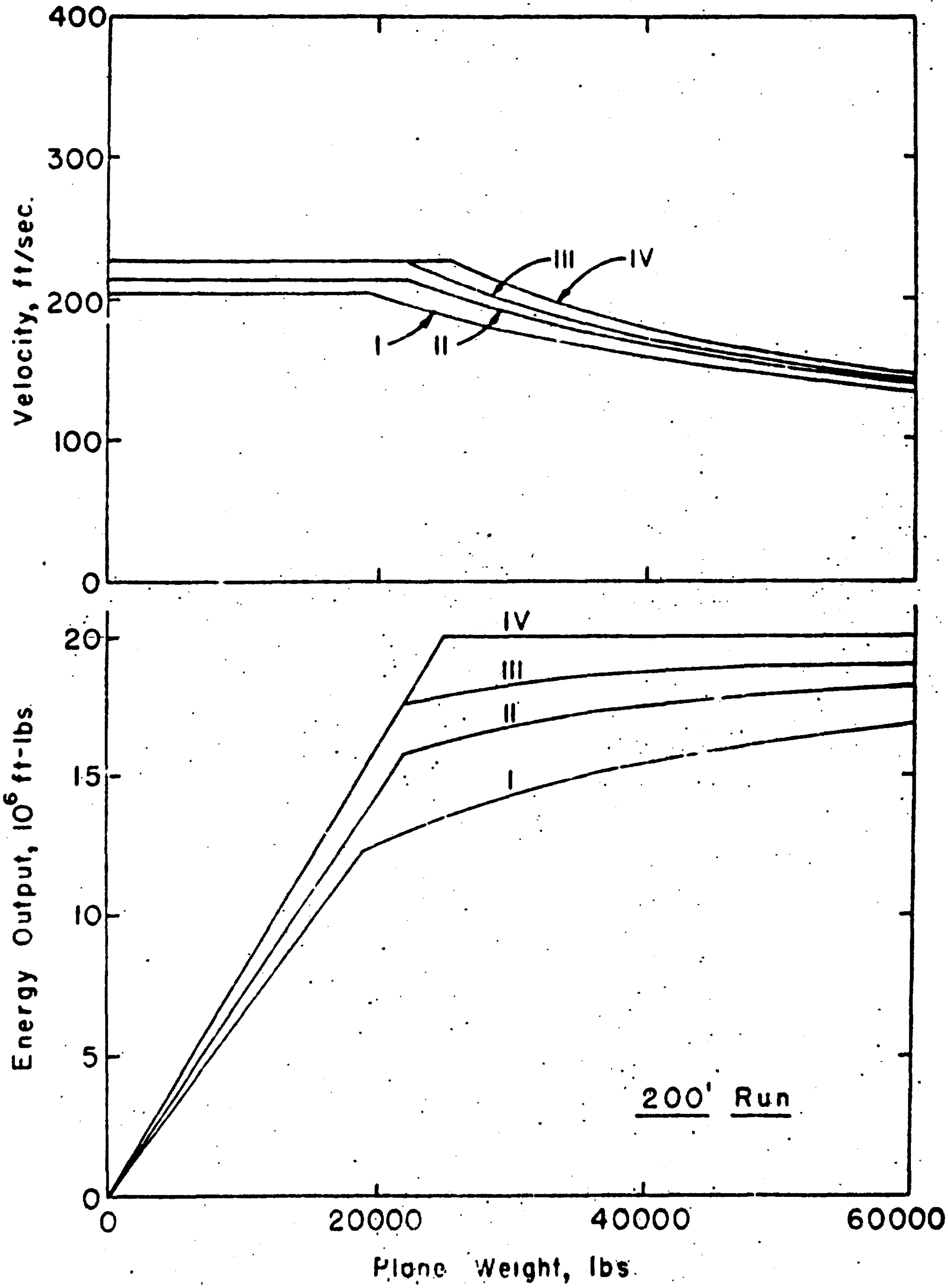


Figure 10. Performance of 200 ft. catapults.

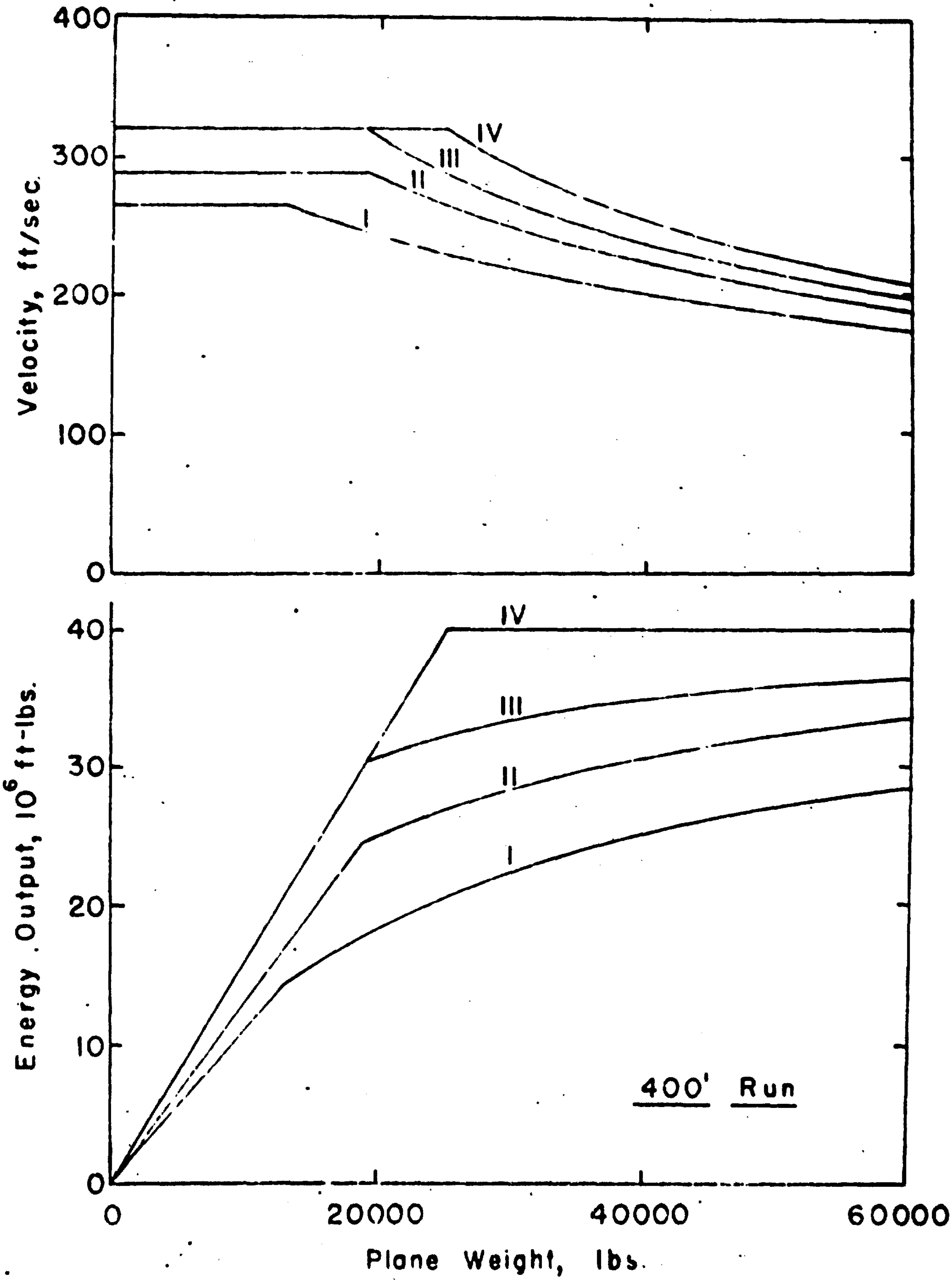


Figure 17. Performance of 400 ft. catapults.

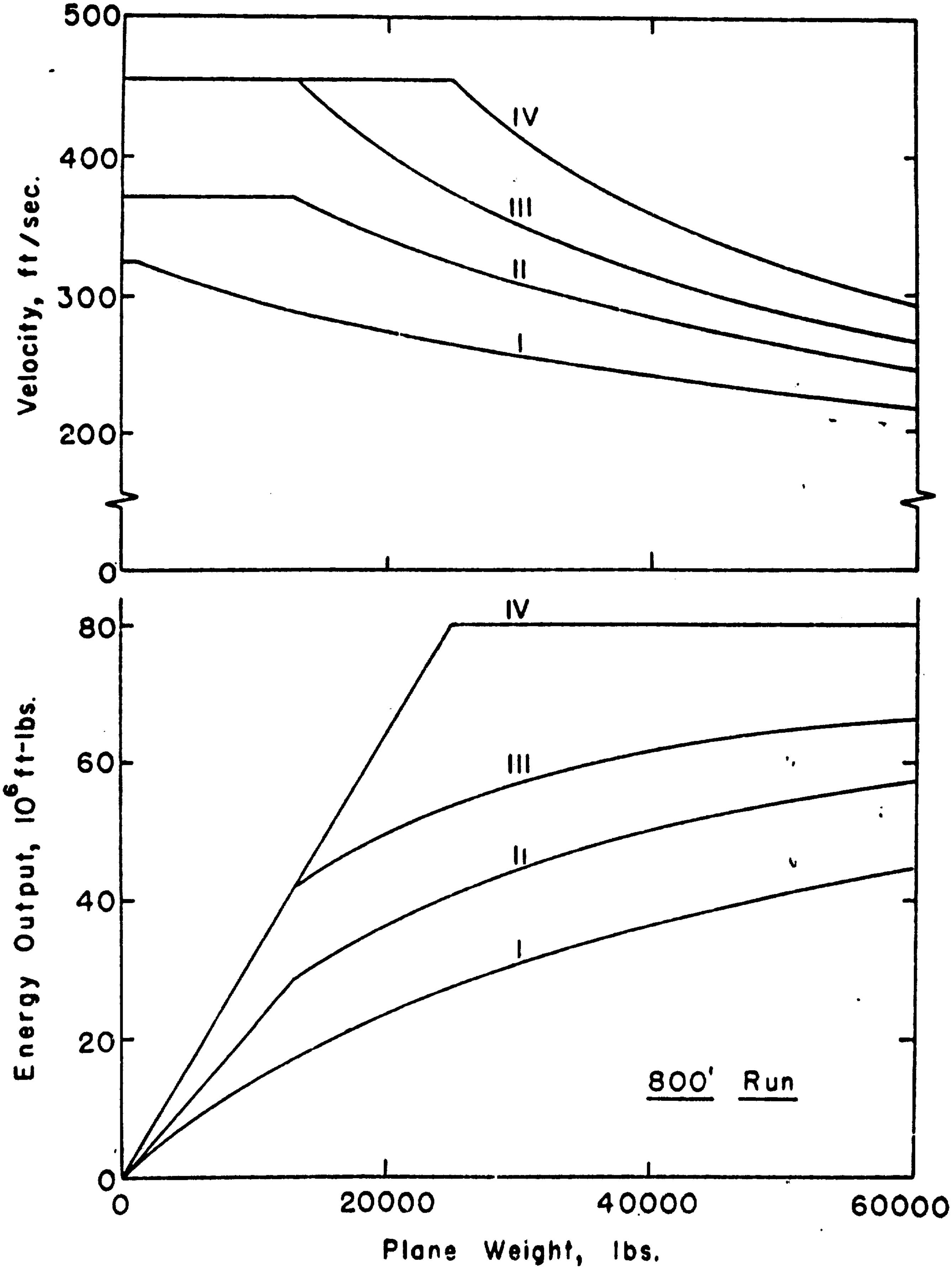


Figure 20. Performance of 800 ft. catapults.

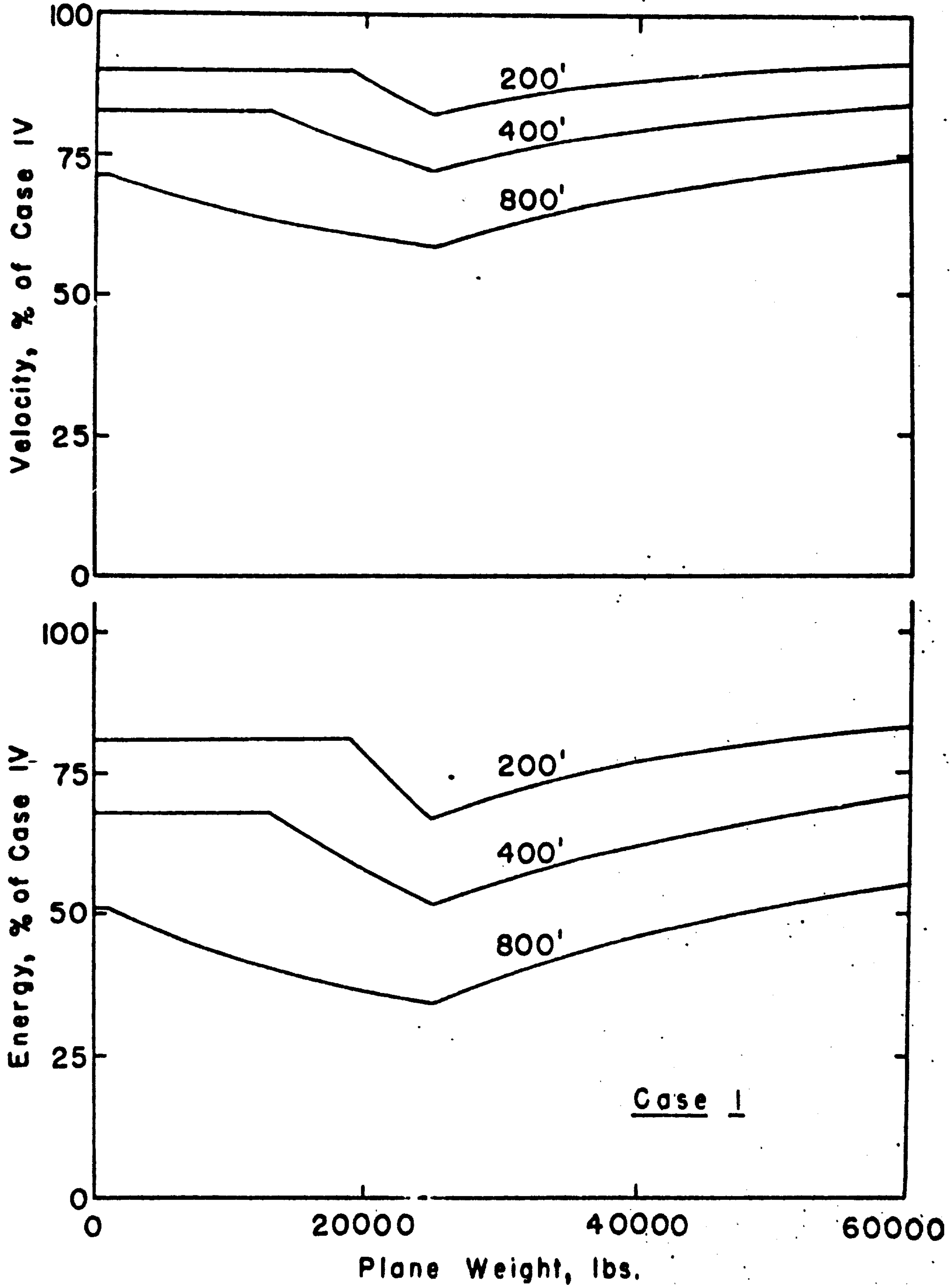


Figure 21. Performance of indirect-drive catapult with retrieving cable braking, as compared to direct-drive.

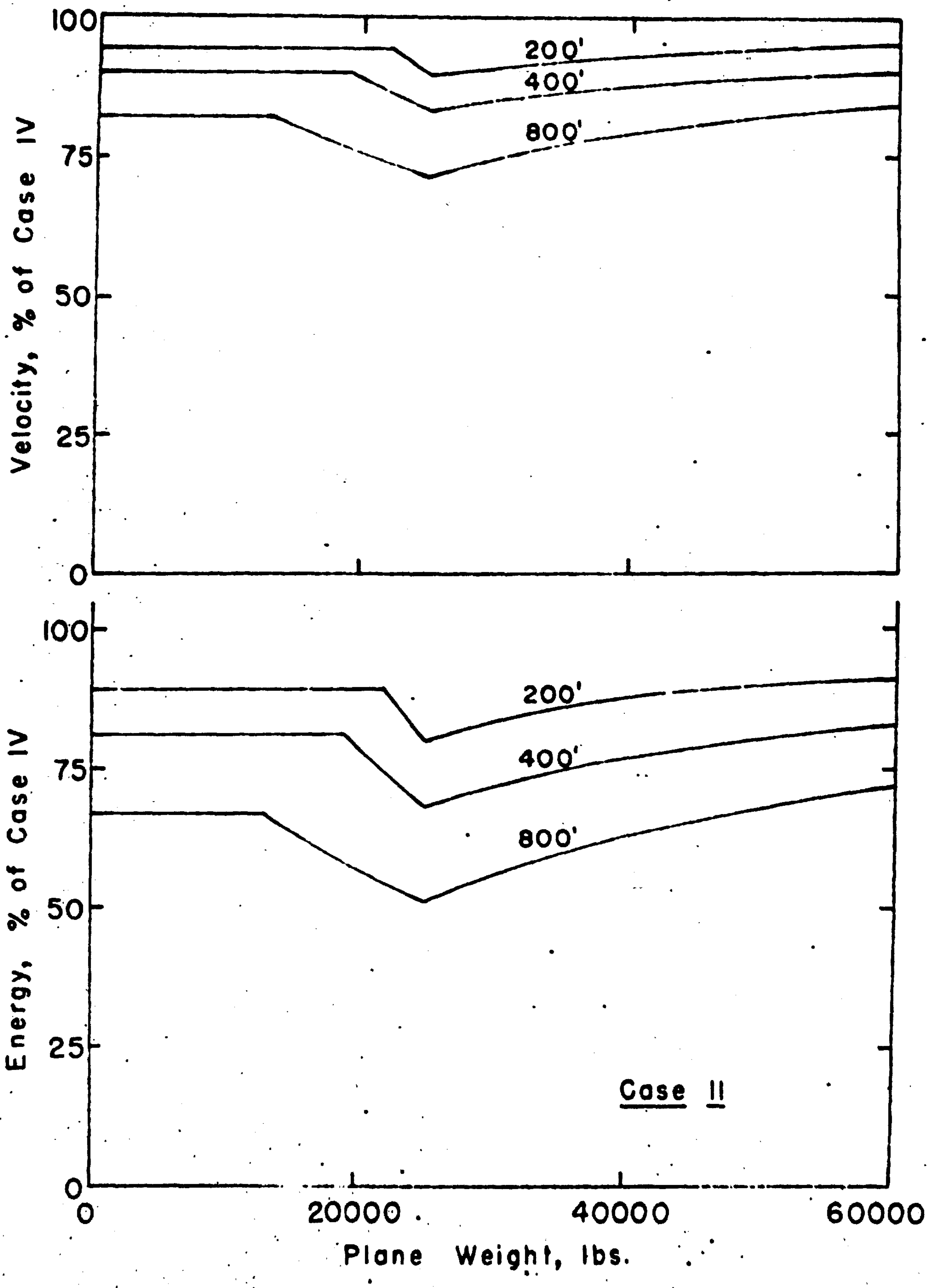


Figure 22. Performance of indirect-drive catapult with shuttle braking, as compared to direct-drive.

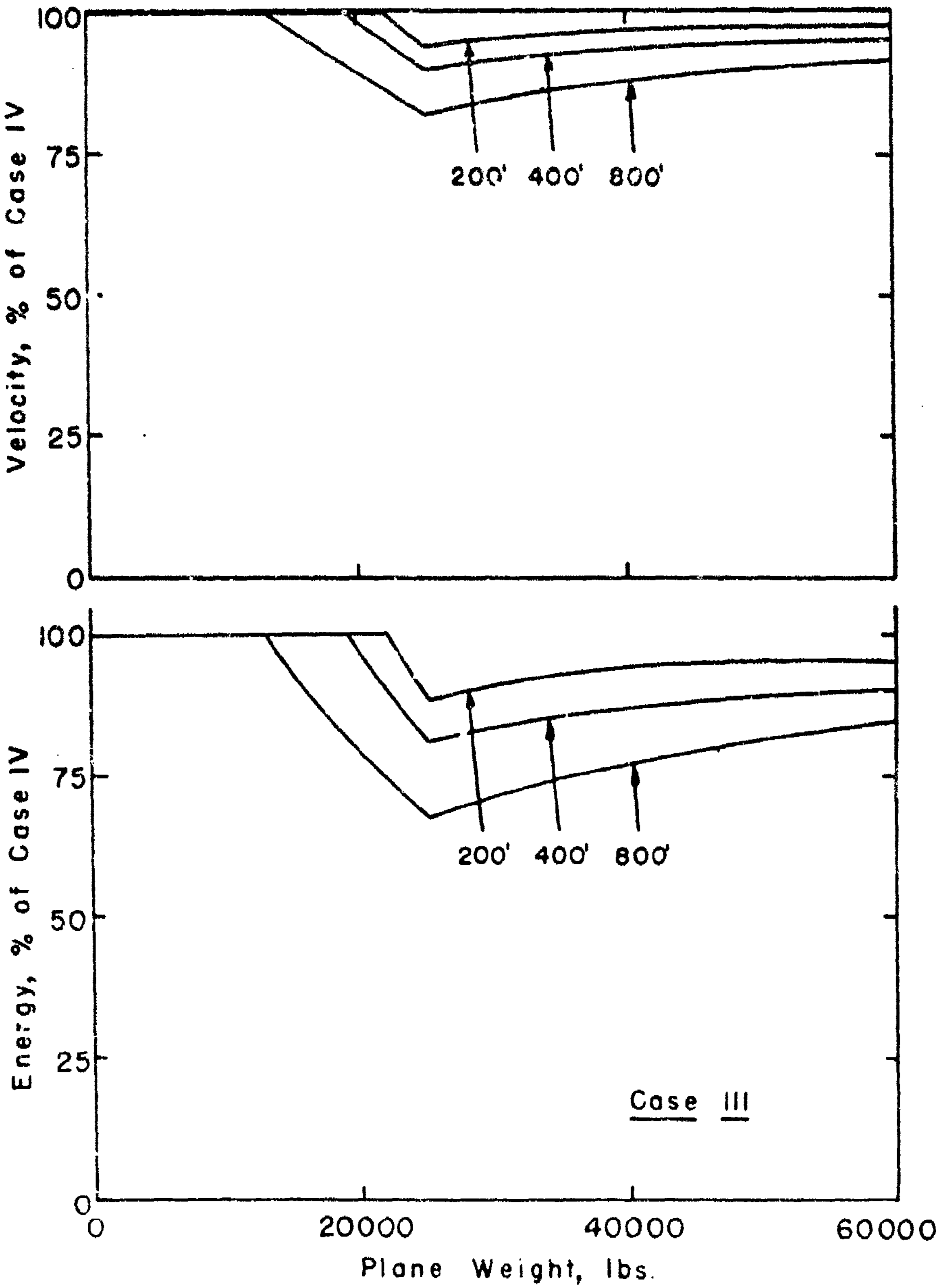


Figure 23. Performance of indirect-drive catapult with multiple braking, as compared to direct-drive.

CONCLUSIONS

The results of the foregoing sections show clearly that limitations are imposed upon the performance of a catapult when the force produced by the engine is transmitted to the aircraft by means of an accelerated cable. The use of the cable imposes a limit on the maximum velocity that can be attained, and reduces the velocity under other operating conditions below that obtainable were the same engine used for a direct-drive system.

The conclusions to be drawn from the previous analyses may be summarized as follows:

1. Indirect-drive cable catapults have an inherent upper speed limit beyond which it is impossible to accelerate without exceeding the safe working strength of the cable. This ultimate limit for a length of conventional cable equal to the total length of run has been shown to be 802 ft/sec.

2. Although this limiting velocity is considerably in excess of any velocity required at present or in the near future, the usable speed that can be obtained in a practical configuration and without excessively heavy cables is considerably less than the ultimate maximum.

3. In terms of idealized cable limitations only, without regard to increases in engine power and total weight introduced by the use of cables, the maximum speed of any catapult is determined by the permissible acceleration and the total available length of run. The speed limitations here discussed, then, do not become prohibitive, except with a high degree of multiple reeving, until either the allowable acceleration or the length of run is increased to a value considerably in excess of that in current use. The practical limit of a cable-drive system arises from the rapid increase in total weight of the installation as the length of the accelerated run is increased to provide a higher terminal velocity. The increase in total weight becomes serious at velocities well below the ideal limit.

4. A multiply reeved system introduces more serious limitations than a 1-to-1 reeved system. The limiting maximum velocity attainable, regardless of weight, decreases as the multiplication of reeving is increased. For high degrees of reeving this decrease results in a limiting velocity not greatly in excess of the velocities to be expected in the immediate future. Even more important, the total weight of an installation necessary to attain a specified velocity even considerably less than the maximum, is appreciably greater for a multiply reeved system than for a singly reeved system. The University of Kansas group believes that multiply reeved systems are impracticable for catapults of even the immediate future.

5. The seriousness of the cable limitations is increased as the amount of cable (or equivalent length of other accelerated and decelerated masses) is increased. For this reason, indirect-drive catapult performance can be improved over the existing hydraulic arrangement by:

   a. The use of shuttle or multiple braking systems;

   b. Elimination of multiple-reeving systems.

   It must be recognized, however, that the elimination of multiple reeving almost necessarily eliminates the hydraulic catapult for high capacity service. It is extremely unlikely that a 1-to-1 coupled indirect-drive hydraulic system could prove feasible, partly from weight and size considerations, but particularly from high velocity fluid flow limitations.

6. The limitations in present systems result in part from the necessity of braking the entire moving cable system by means of forces transmitted through the retrieving cable. For this reason the length of accelerated run is considerably less than the total length of run available. Thus the suggested improvements in braking arrangements (i.e. shuttle or multiple braking, as discussed on page 38), improve performance not only by reducing the weight of accelerated cable but also by increasing the ratio of accelerated to total run.

7. There is no limit to the weight of aircraft which can be launched with cable-drive catapults. Furthermore, the use of cable materials of higher strength to weight ratio, which may soon become commercially available, will increase the usefulness of this type of unit. Finally, although the indirect-drive cable catapult is not capable of indefinite extension to ever higher velocities, the 1-to-1 indirect cable drive is capable of extension to velocities sufficiently higher than those now used that it will be practicable for several years to come. It should be realized, however, that the range of extrapolation to very high capacity is limited for all indirect-drive cable catapults, and emphasis must be placed on the development of other types.

8. The continuous cable drive, shuttle clutch catapult offers possibilities for medium capacity installations. Such a unit retains many of the equipment-location advantages of the indirect-drive cable catapult, but is without several of the severe disadvantages. On the other hand, the introduction of the shuttle clutch leads to different specific difficulties of design, construction and service. The shuttle clutch catapult is almost certainly impractical for high capacity installations.

[redacted]

9. In the final analysis, the choice of a catapult is a compromise among the contradictory desires for the best possible unit from the standpoints of:- (a) low over-all weight; (b) low topside weight; (c) high efficiency; (d) protection from damage; (e) reliability and durability; (f) lack of interference with the armor and structure of the flight deck and with other carrier equipment. The indirect-drive cable catapult appears quite favorable at present in all of these respects except over-all weight and efficiency. For low and medium capacity use, it is satisfactory in its present stage of development. However, since it has definite limitations even within the range of speeds possible in the foreseeable future, it seems futile to expend further great effort on its development for very high capacity use. Rather, development work should be concentrated on the more practical direct-drive systems capable of much higher capacities. If space and convenience dictate, any direct-drive unit can be converted to an indirect-drive system of smaller capacity. Thus if successful high capacity direct-drive units can be developed, the whole launching problem is solved.

[redacted]